Dynamics of electric systems

Robert Beloiu

University of Pitesti, Romania

robertbeloiu@gmail.com

The transfer function

- A linear and invariant system is described by the equation:

$$a_0 * \overset{(n)}{y} + a_1 * \overset{(n-1)}{y} + \ldots + a_{n-1} * \overset{(\bullet)}{y} + a_n * y =$$
$$= b_0 * \overset{(m)}{x} + b_1 * \overset{(m-1)}{x} + \ldots + b_{m-1} * \overset{(\bullet)}{x} + b_m * x, (n \geq m)$$

- where:

The transfer function

- $G(s) = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[x(t)]} = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s^1 + b_m s}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s^1 + a_n s}, n \geq m$

$$b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \ldots = const. * (s - s_{z_1}) * (s - s_{z2}) * \ldots * (s - s_{zm})$$

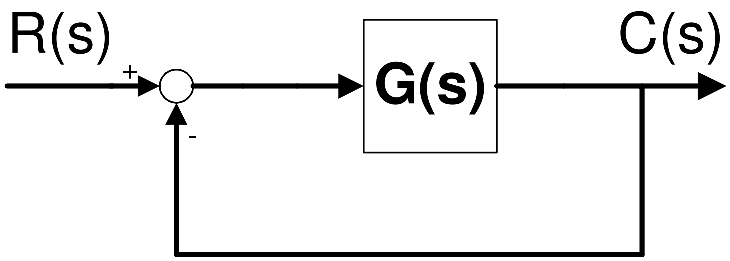
$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots = const. * (s - s_1) * (s - s_2) * \ldots * (s - s_n)$$

The transfer function

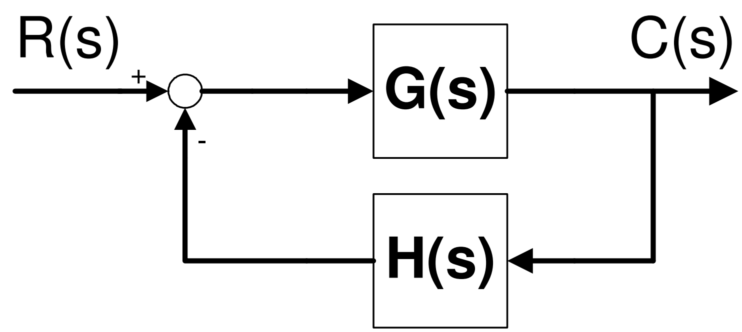
$$G(s) = ct * \frac{(s - s_{z_1}) * (s - s_{z2}) * \ldots * (s - s_{zm})}{(s - s_1) * (s - s_2) * \ldots * (s - s_n)} \quad (n \geq m)$$

- **<u>Zeros</u>**.
- **<u>Poles</u>**.

Block schematic

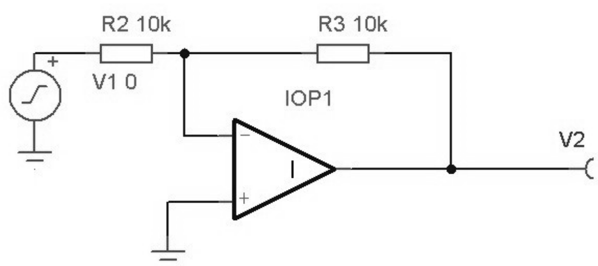


Unity feed-back



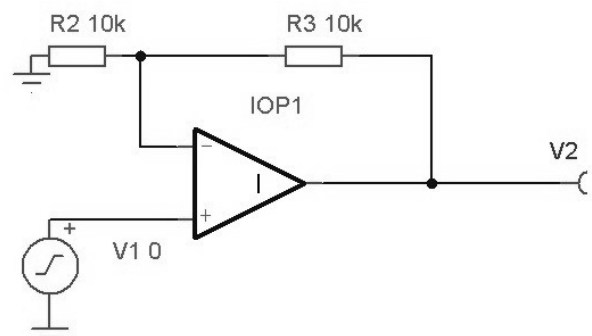
Non-unity feed-back

Circuits with OA: basic structures

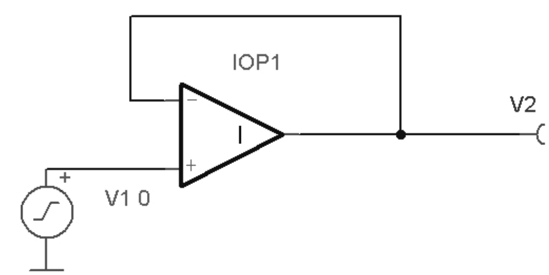


Inverting

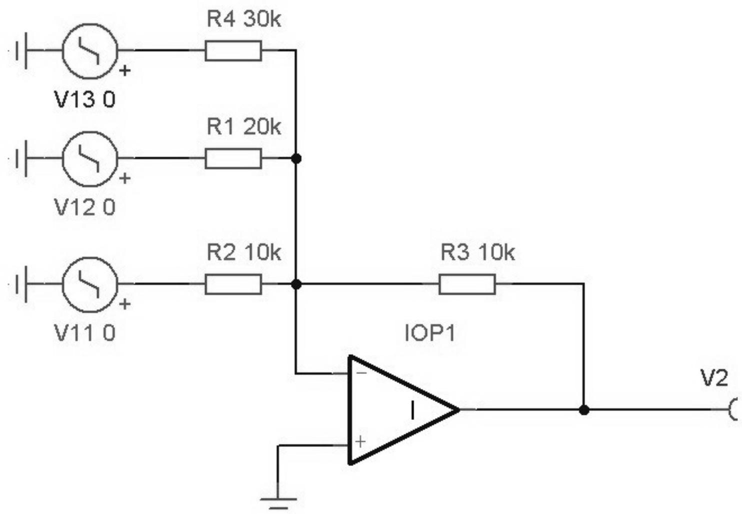
Non-inverting

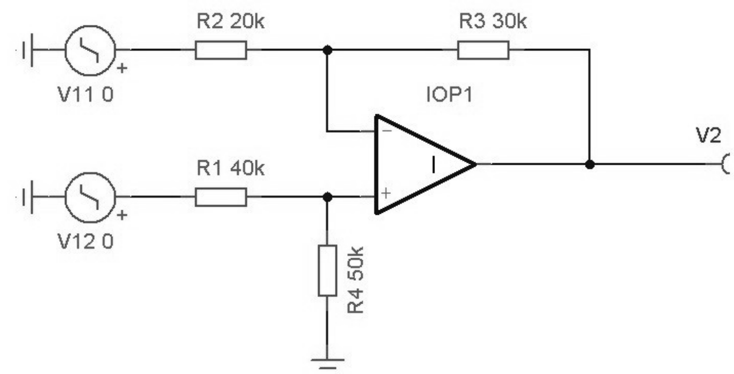


Repeating

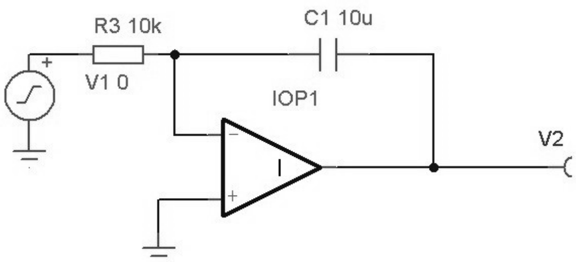


Sum

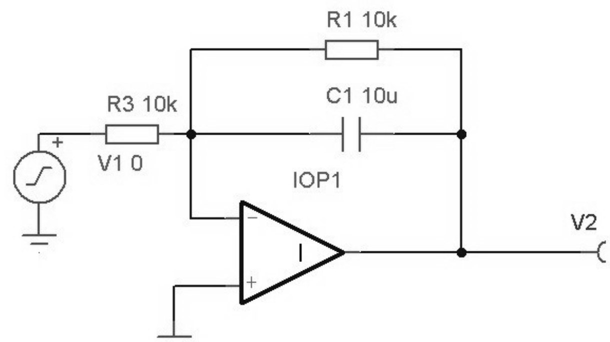

R4 30k
V13 0
R1 20k
V12 0
R2 10k
R3 10k
V11 0
IOP1
V2


R2 20k
R3 30k
V11 0
IOP1
V2
R1 40k
V12 0
R4 50k

Derivation and integration

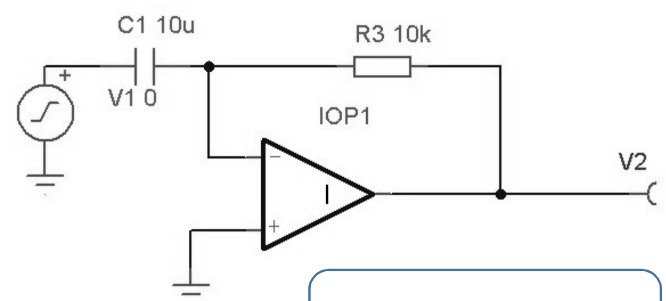


Derivation



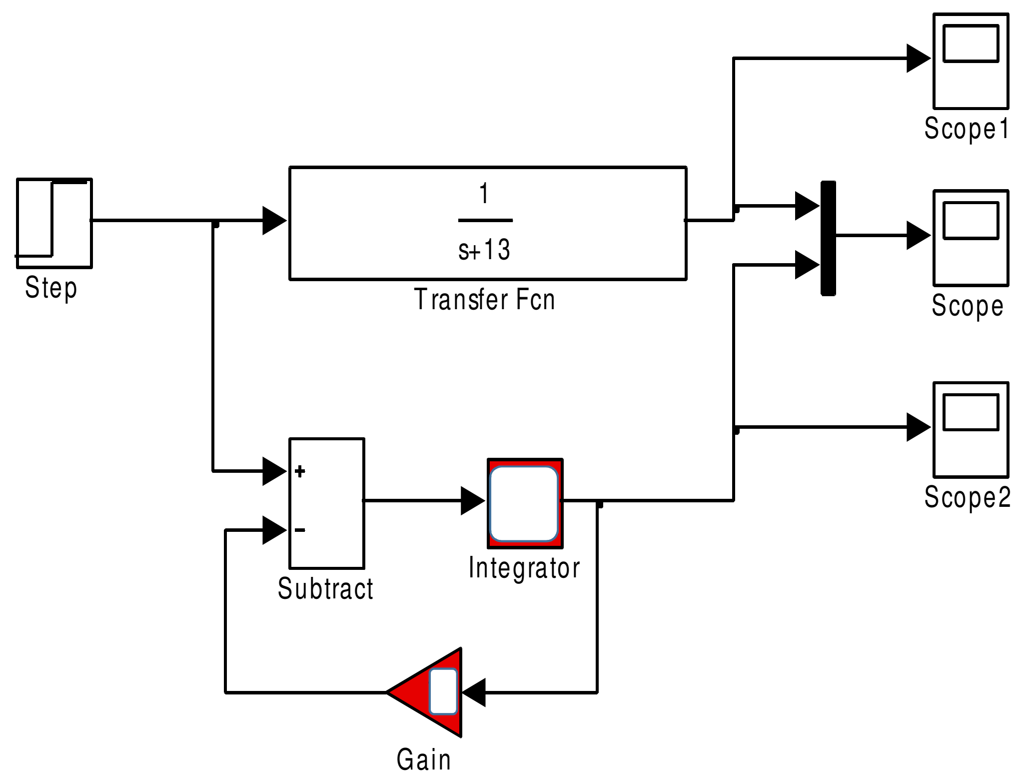
Derivation

Integration



Equivalent schematics

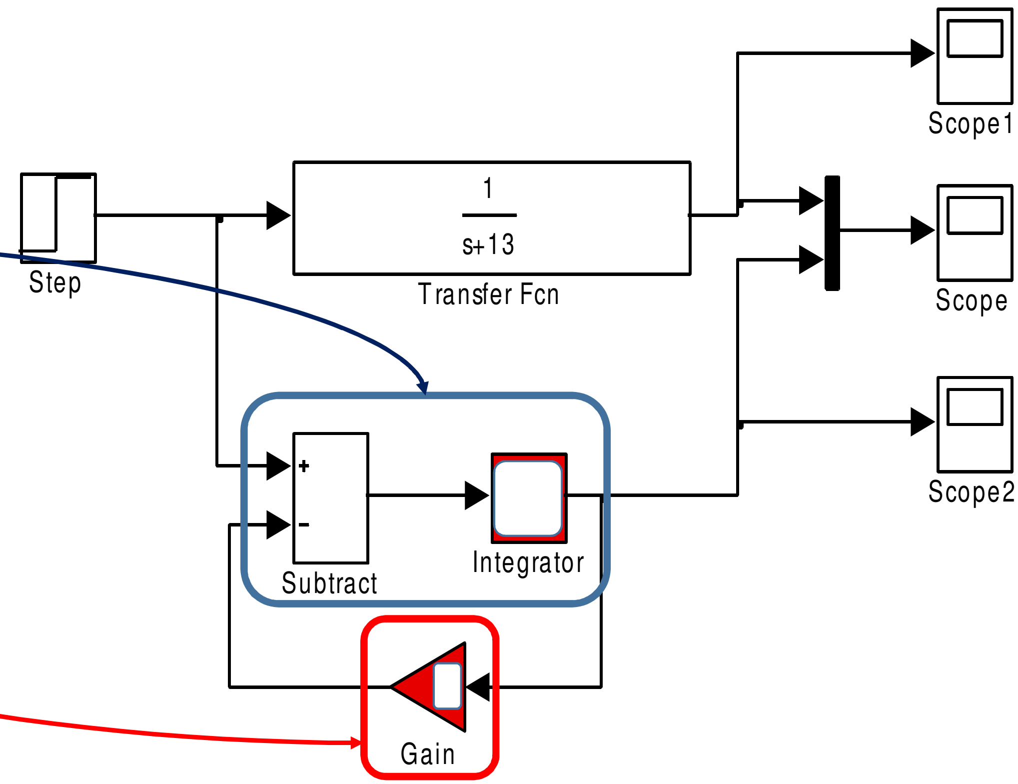
$$G(s) = \frac{1}{s+13} = \frac{\frac{1}{s}}{1 + 13 \cdot \frac{1}{s}}$$



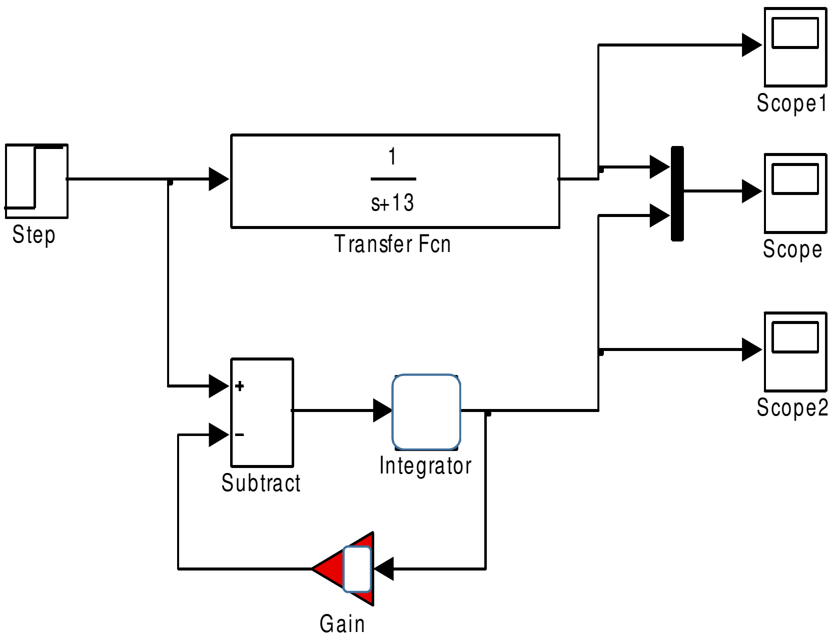
Equivalent schematics

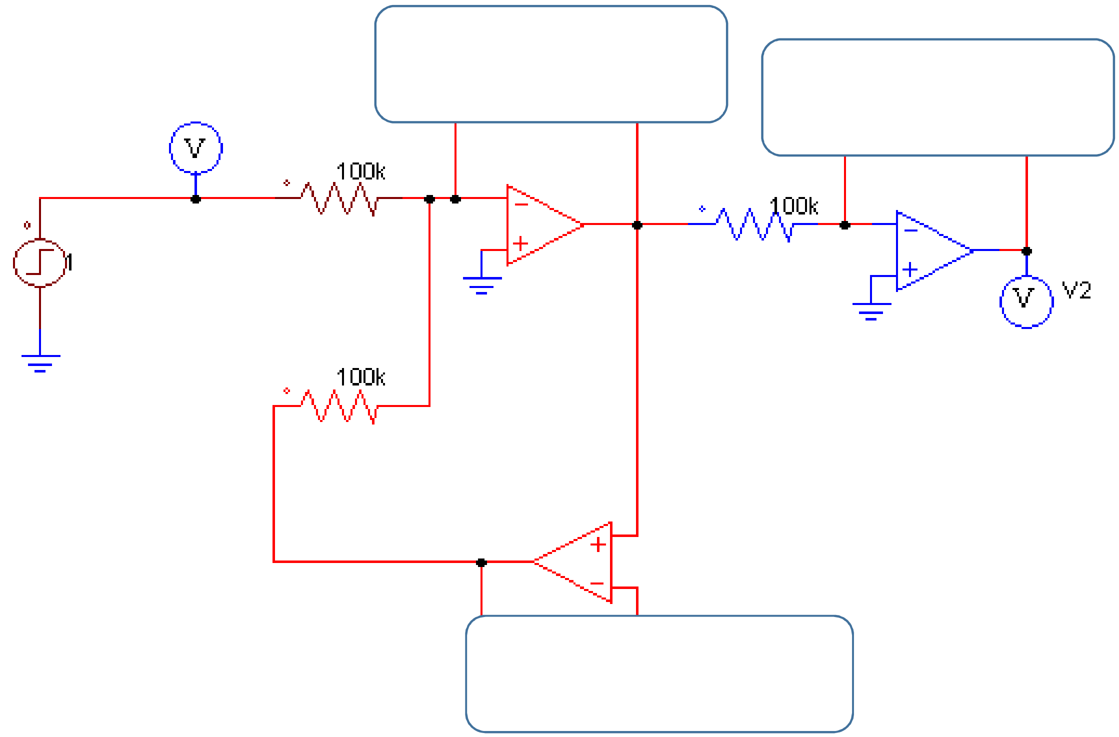
$$G(s) = \frac{1}{s+13} = \frac{\frac{1}{s}}{1+13\cdot\frac{1}{s}}$$

$$G(s) = \frac{1}{s+13} = \frac{\boxed{\frac{1}{s}}}{1+\boxed{13}\cdot\boxed{\frac{1}{s}}}$$

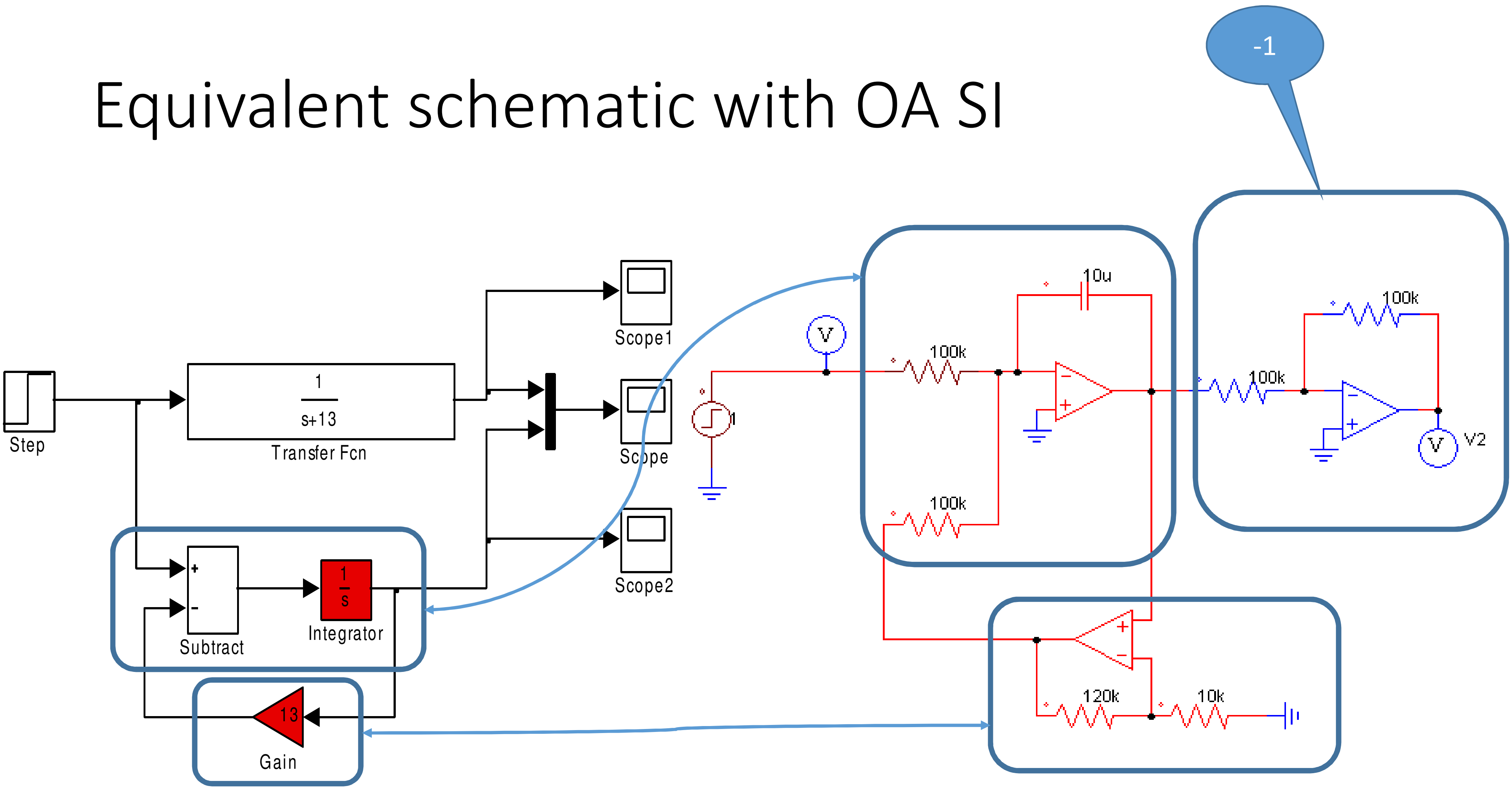


Equivalent schematic with OA





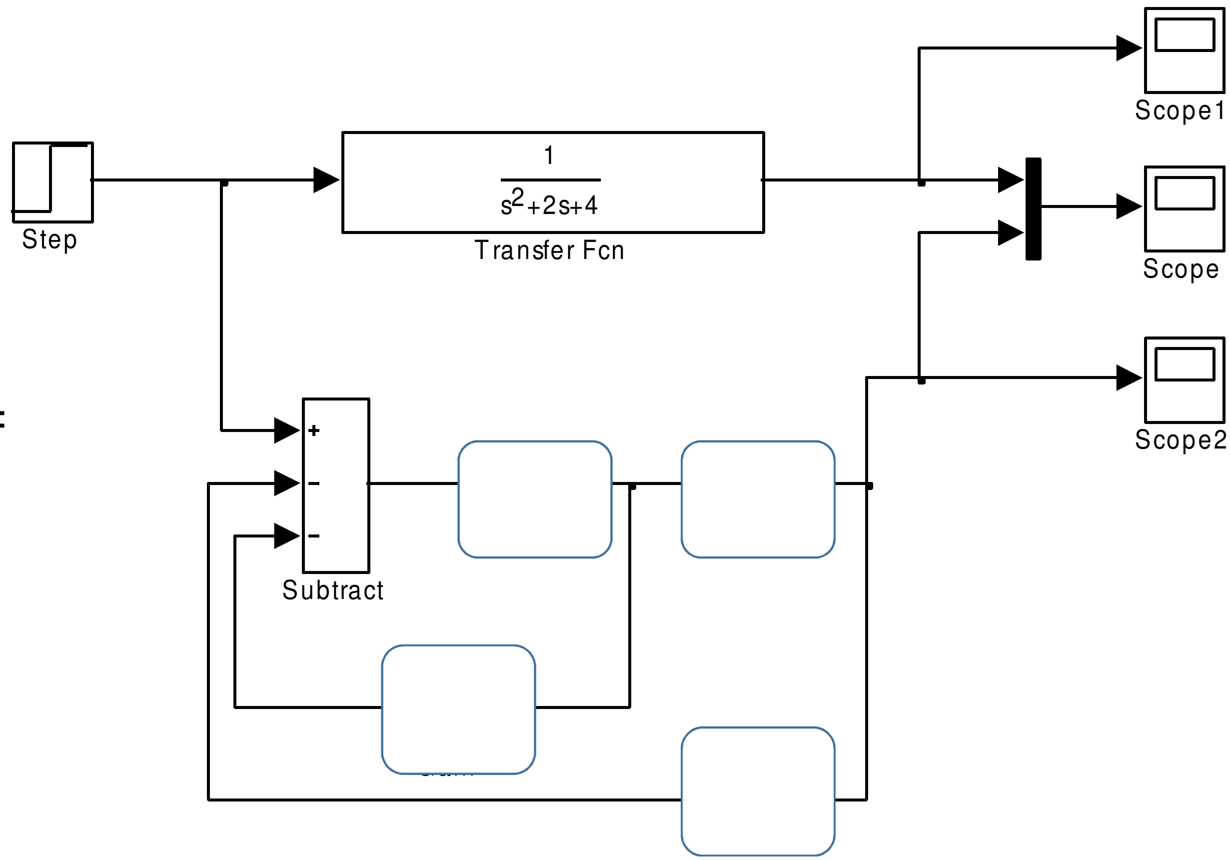
Equivalent schematic with OA SI



Equivalent schematic with OA SII

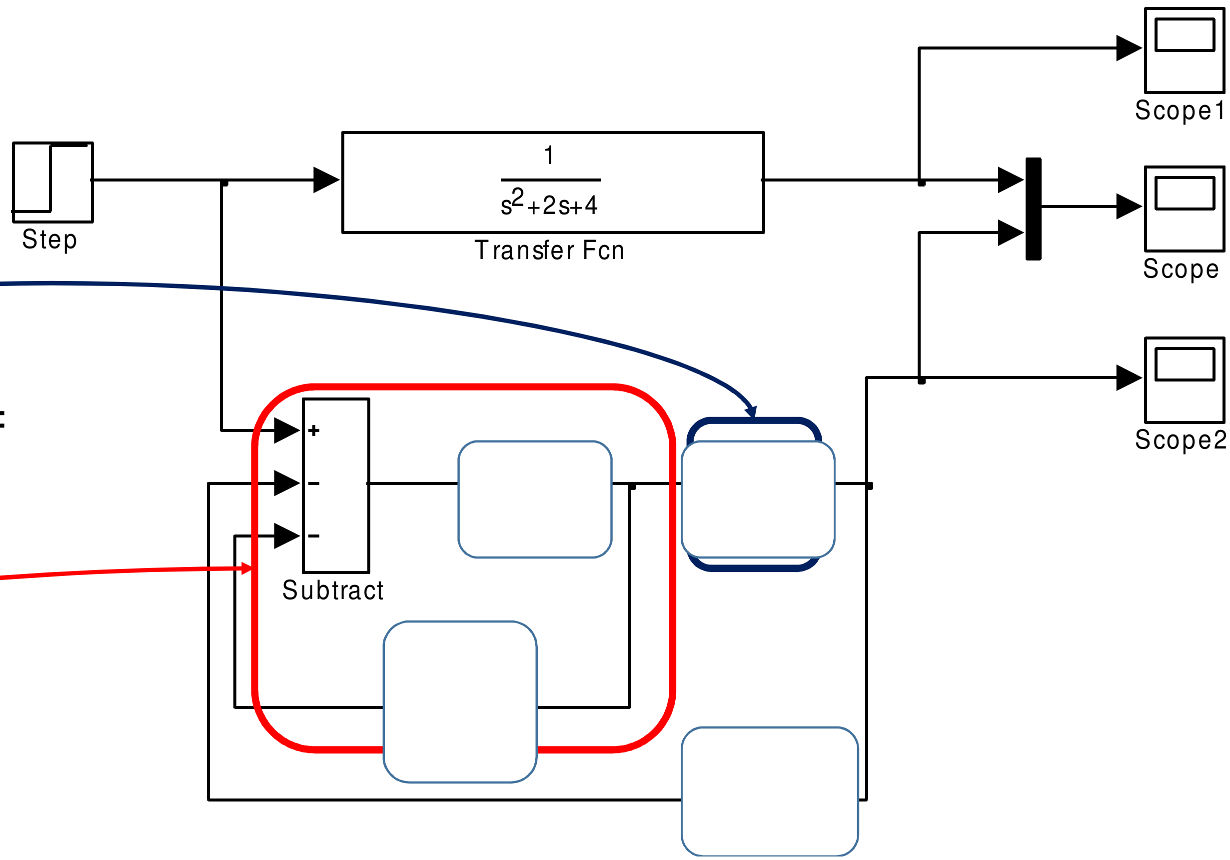
- $G(s) = \frac{1}{s^2+2\cdot s+4} =$
- $= \frac{\frac{1}{s^2+2\cdot s}}{1+4\cdot\frac{1}{s^2+2\cdot s}}$

- $G_1(s) = \frac{1}{s^2+2\cdot s} = \frac{1}{s} \cdot \frac{1}{s+2} =$
- $= \frac{1}{s} \cdot \frac{\frac{1}{s}}{1+\frac{2}{s}}$

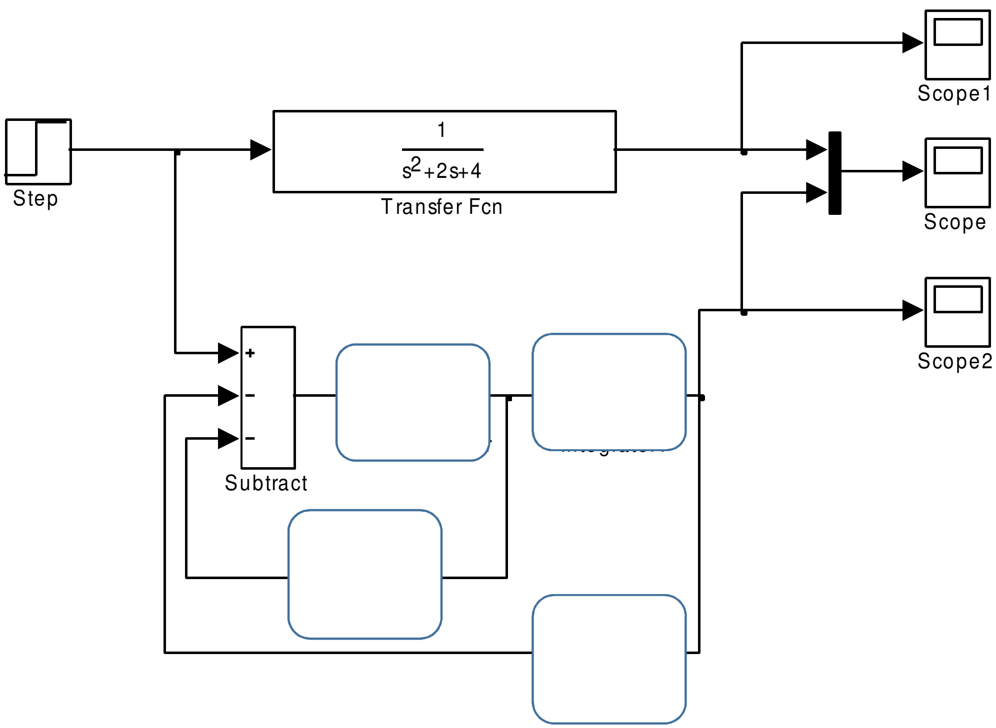


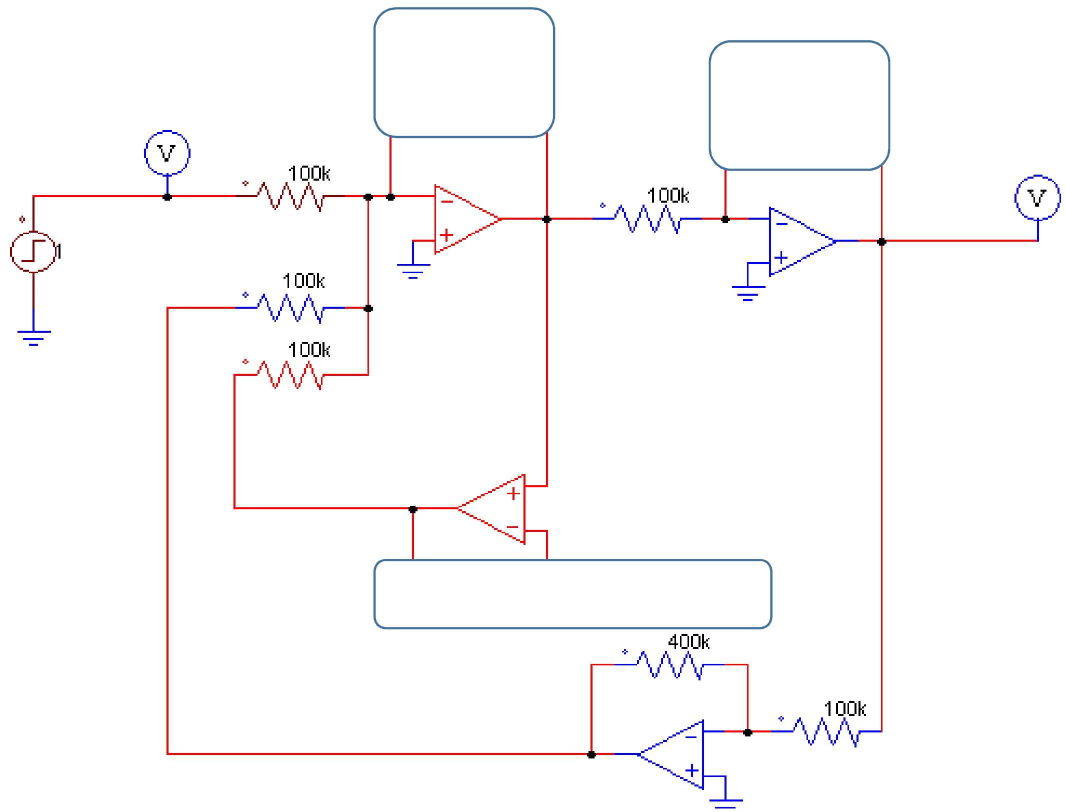
Equivalent schematic with OA

- $G(s) = \dfrac{1}{s^2+\boxed{2}\,s+\boxed{4}} =$
- $= \dfrac{\frac{1}{s^2+\boxed{2}\cdot s}}{1+\boxed{4}\cdot\frac{1}{s^2+\boxed{2}\cdot s}}$
- $G_1(s) = \dfrac{1}{s^2+2\cdot s} = \dfrac{1}{s}\cdot\dfrac{1}{s+2} =$
- $= \boxed{\dfrac{1}{s}}\;\boxed{\dfrac{\frac{1}{s}}{1+\frac{2}{s}}}$

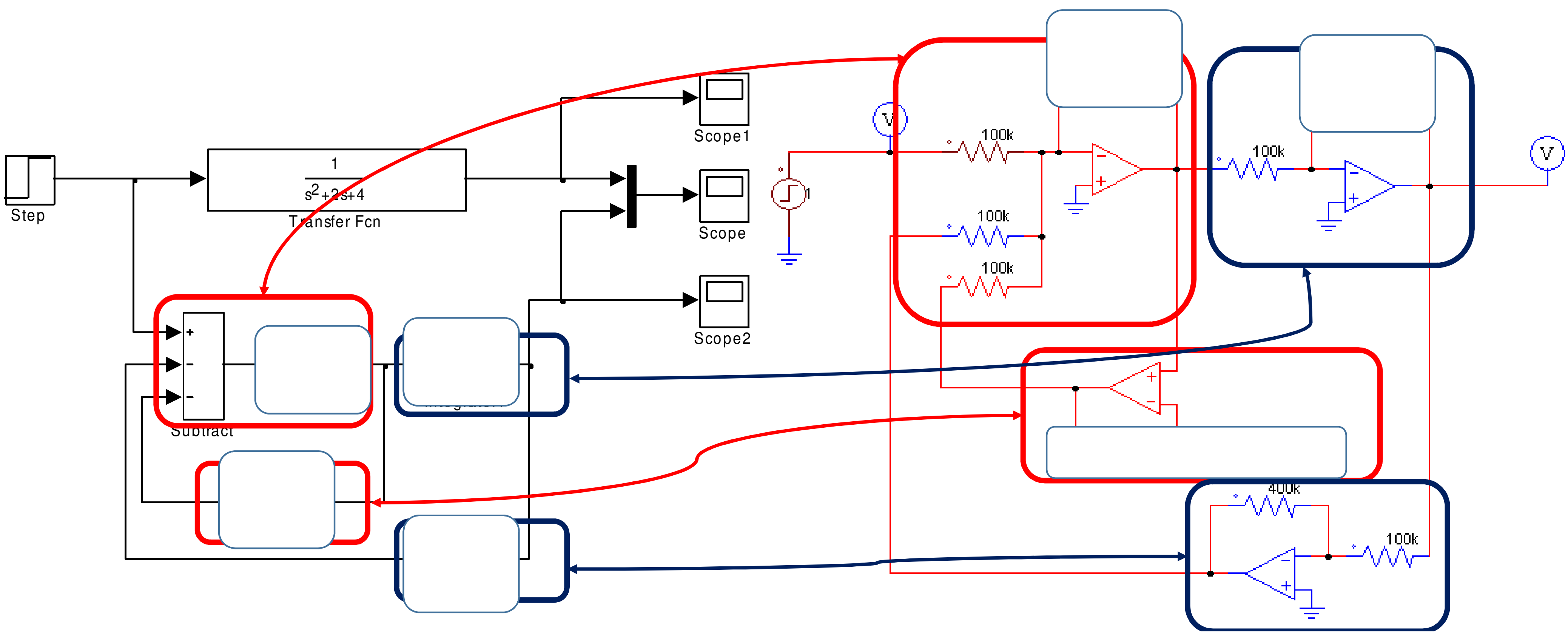


Equivalent schematic with OA

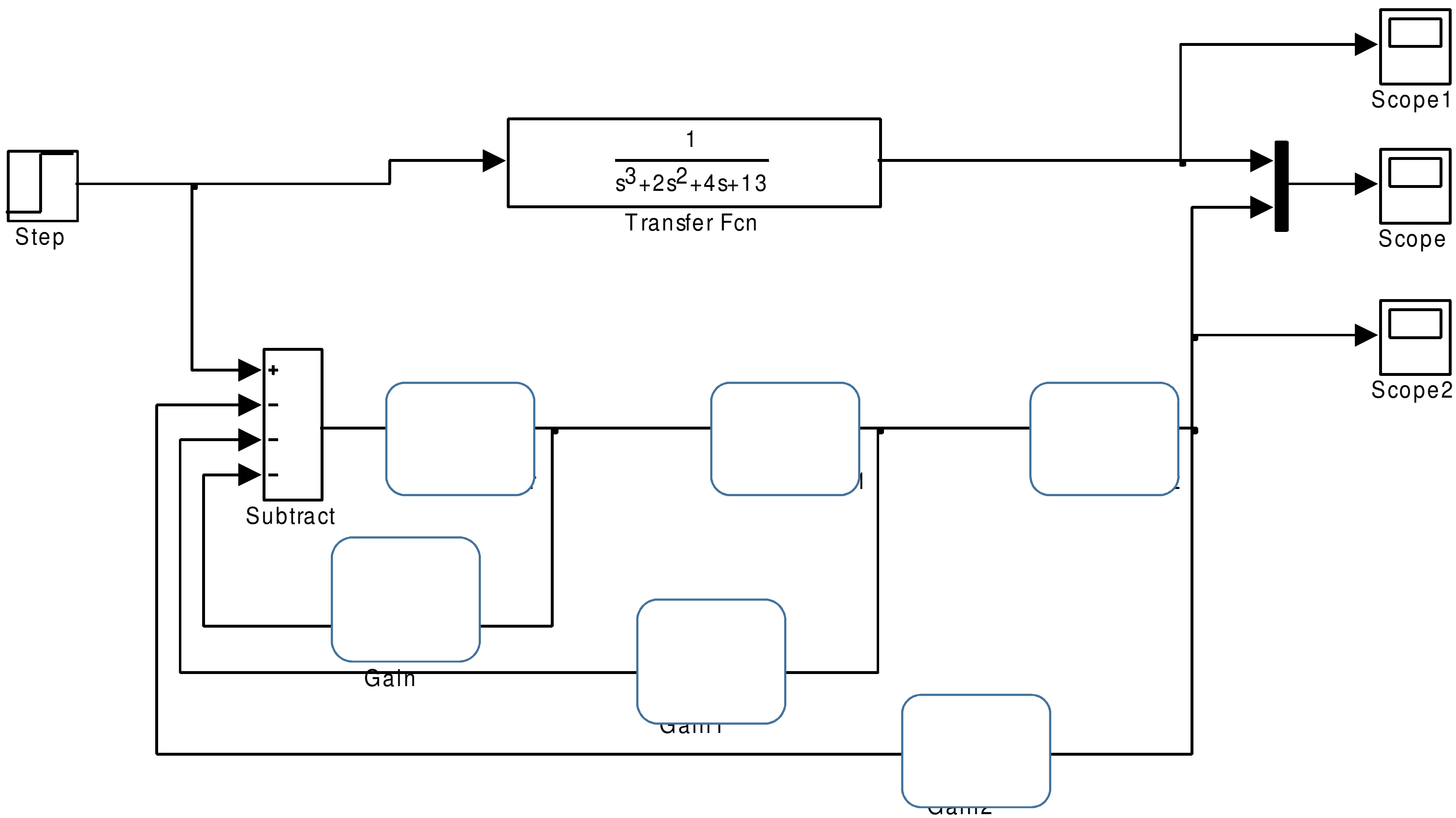




Equivalent schematic with OA



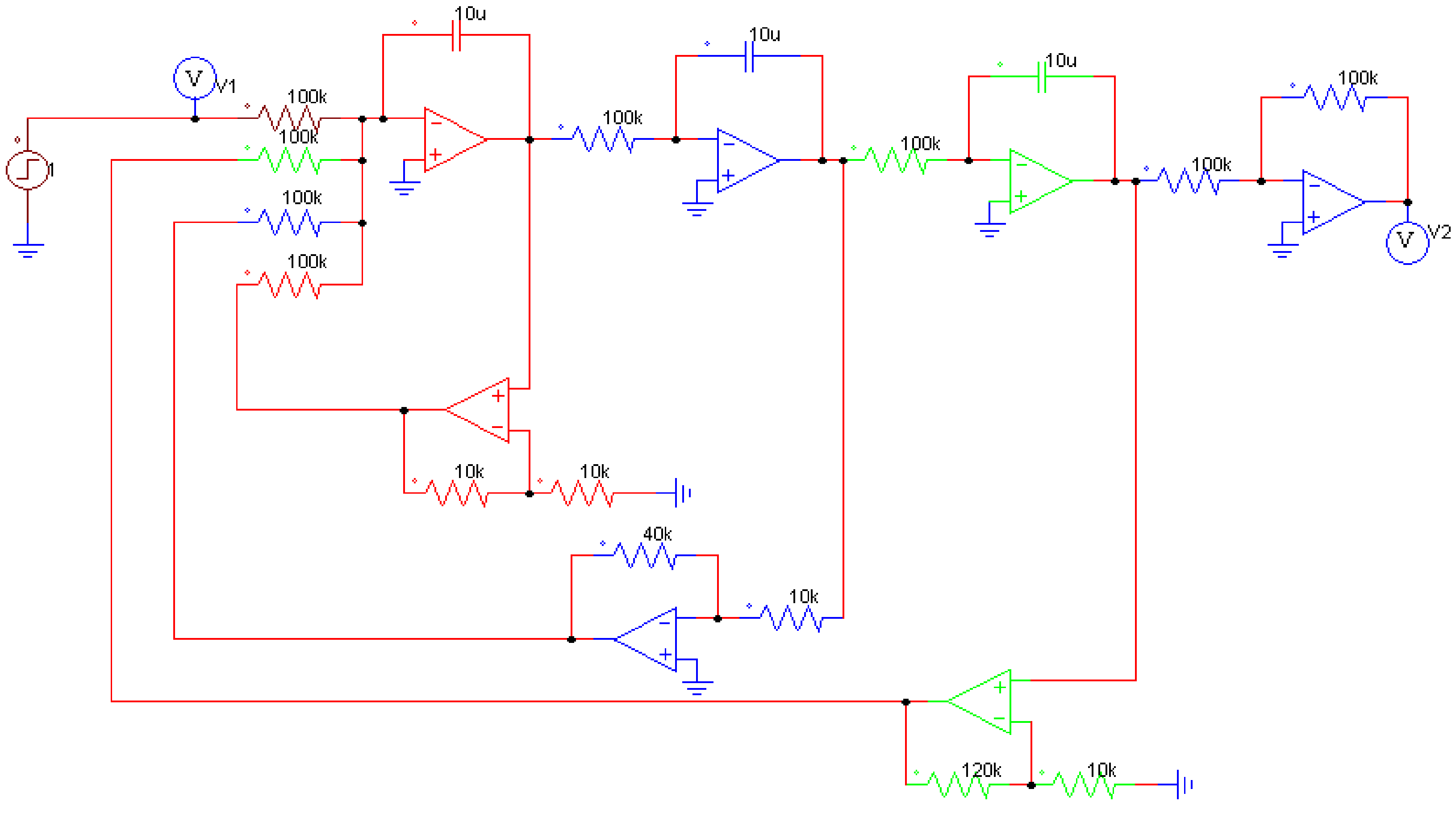
Equivalent schematic with OA



Equivalent schematic with OA

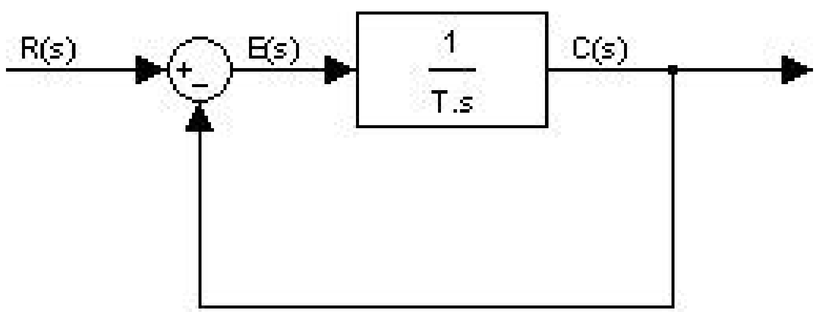
- $G(s) = \frac{1}{s^3+2\cdot s^2+4\cdot s+13} = \frac{\boxed{\frac{1}{s^3+2\cdot s^2+4\cdot s}}}{1+13\cdot\boxed{\frac{1}{s^3+2\cdot s^2+4\cdot s}}}$
-
- $G_1(s) = \boxed{\frac{1}{s^3+2\cdot s^2+4\cdot s}} = \frac{1}{s}\cdot\frac{1}{s^2+2\cdot s+4} = \frac{1}{s}\cdot\frac{\boxed{\frac{1}{s^2+2\cdot s}}}{1+4\cdot\boxed{\frac{1}{s^2+2\cdot s}}}$
- $G_2(s) = \boxed{\frac{1}{s^2+2\cdot s}} = \frac{1}{s}\cdot\frac{1}{s+2} = \frac{1}{s}\cdot\frac{\frac{1}{s}}{1+\frac{2}{s}}$

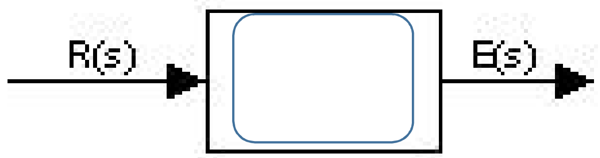
Equivalent schematic with OA



First order system S-I

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)*H(s)} \qquad \frac{C(s)}{R(s)}=\boxed{}$$





First order system S-I: step input

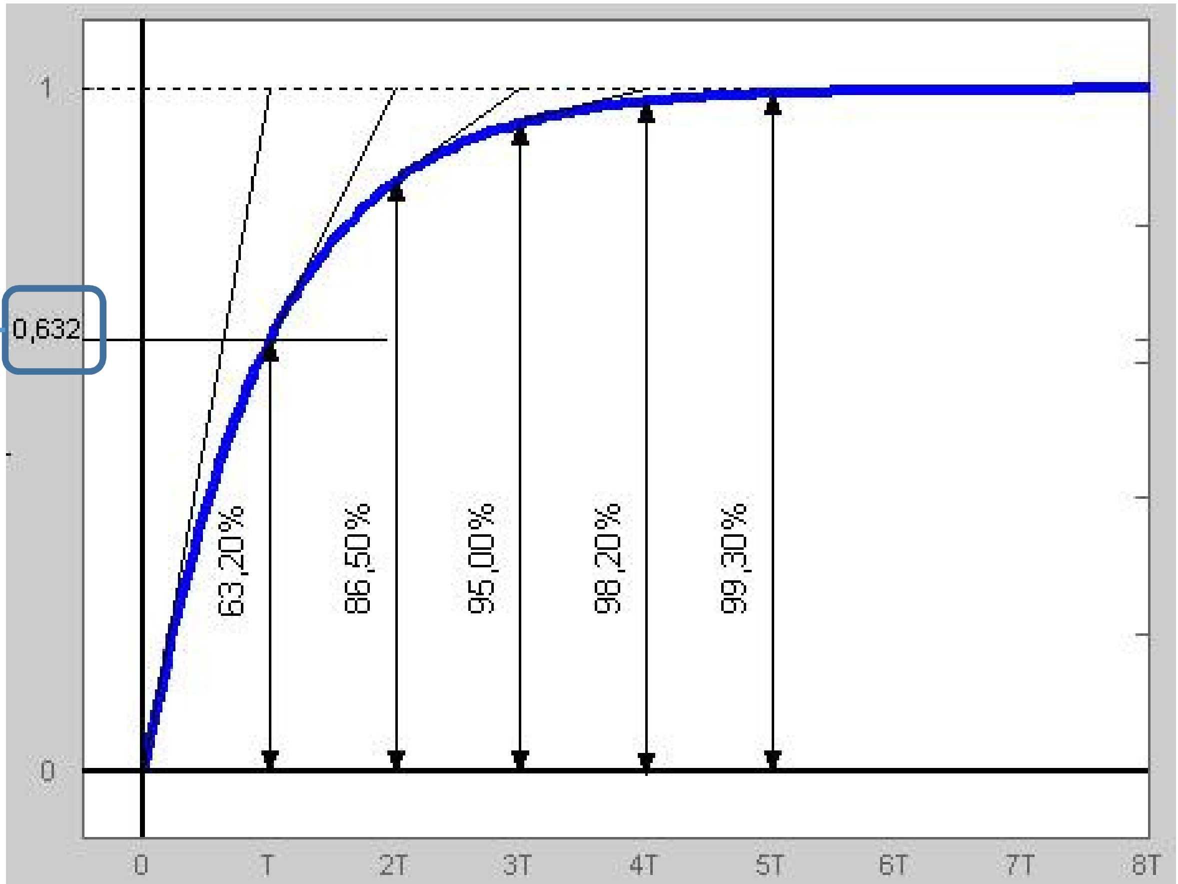
$$c(t) = 1 - e^{-\frac{t}{T}}$$

for t > 0

$$t = T \rightarrow$$

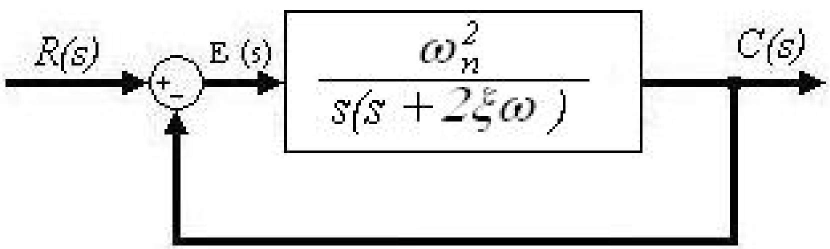
$$c(T) = 1 - e^{-1} =$$

$$=$$



Second order system S-II

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) * H(s)}$$



$$\frac{C(s)}{R(s)} = \boxed{}$$

$$\frac{C(s)}{R(s)} = \boxed{}$$

ζ – factor de amortizare

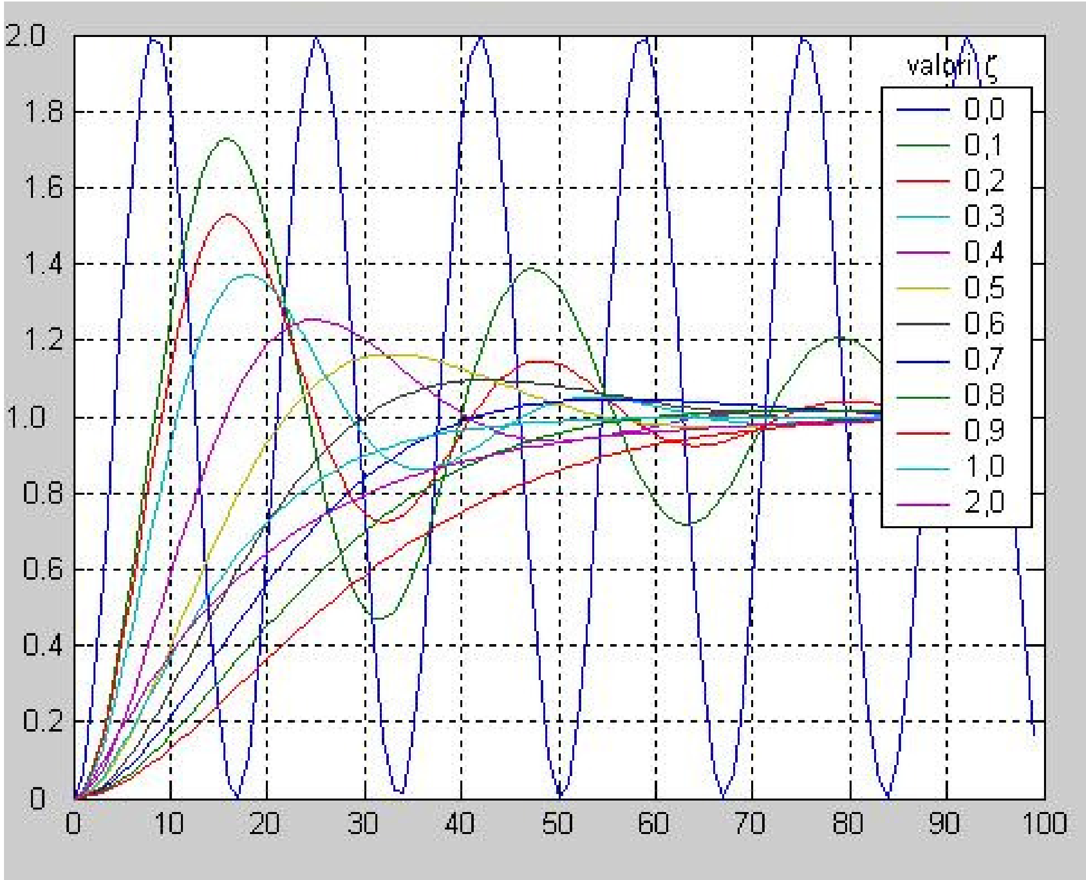
ω_n- pulsatie naturala de oscilatie

Second order system response: step input

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) =$$

$$= 1 - \frac{\xi}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \operatorname{arctg}\frac{\sqrt{1-\xi^2}}{\xi}\right), \qquad t \geq 0$$

Second order system response: step input



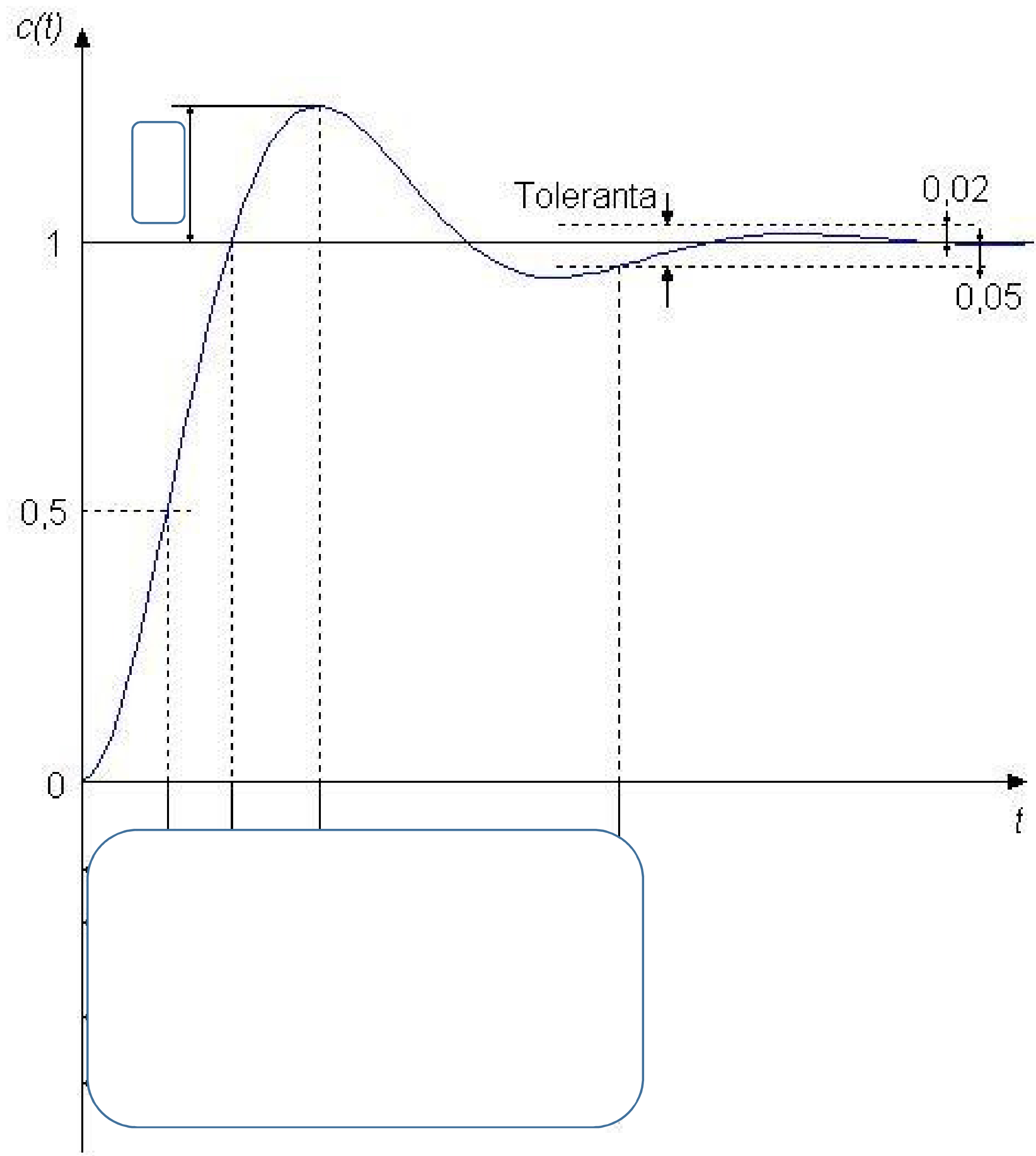
Transitory response: step input

t_d- delay time

t_r- rise time

t_p- peak time σ_p- peak value

t_s- settling time



Determination of transient parameters

Rising time t_r :

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) =$$

$$= 1 - \frac{\xi}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \operatorname{arctg}\frac{\sqrt{1-\xi^2}}{\xi}\right), \qquad t \geq 0$$

$$c(t_r) = 1 - e^{-\xi\omega_n t_r}\left(\mathbf{cos}\,\omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}}\mathbf{sin}\,\omega_d t_r\right)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

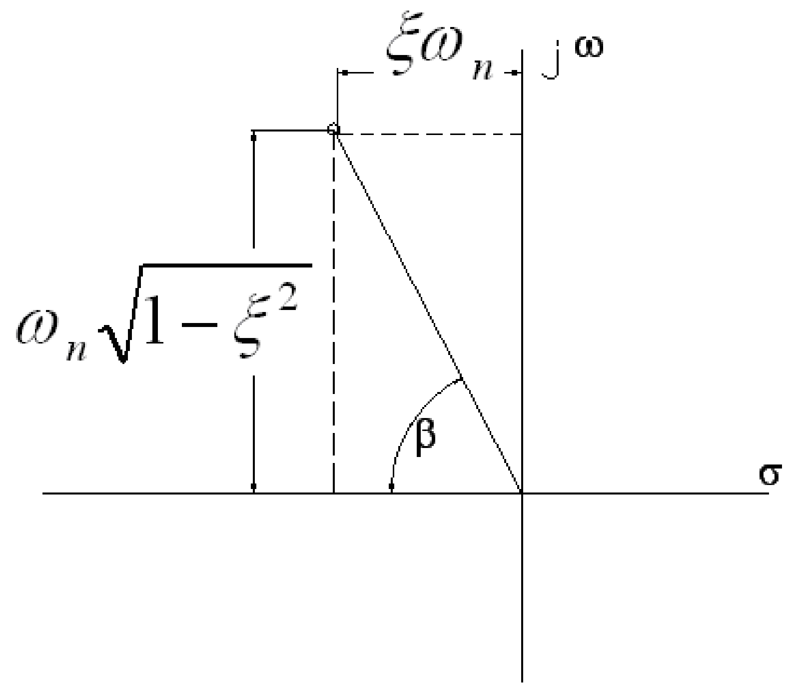
Determination of transient parameters

Rising time t_r :

$$\cos\omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t_r = 0$$

$$\boxed{t_r = \frac{1}{\omega_d}\operatorname{arctg}\left(\frac{\omega_d}{\sigma}\right) = \frac{\pi-\beta}{\omega_d}}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$



Determination of transient parameters

Overshooting time t_p :

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) =$$

$$= 1 - \frac{\xi}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \operatorname{arctg}\frac{\sqrt{1-\xi^2}}{\xi}\right), \qquad t \geq 0$$

Determination of transient parameters

Overshooting time t_p :

$$\left.\frac{dc}{dt}\right|_{t=t_p} = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin \omega_d t_p = 0 \Rightarrow$$

$$t_p = \boxed{}$$

Determination of transient parameters

Overshooting value σ_p :

$$t_p = \frac{\pi}{\omega_d}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right) =$$

$$= 1 - \frac{\xi}{\sqrt{1-\xi^2}}\sin\left(\omega_d t + \operatorname{arctg}\frac{\sqrt{1-\xi^2}}{\xi}\right), \qquad t \geq 0$$

Determination of transient parameters

Overshooting value σ_p :

$\sigma_p =$ $\boxed{}$

$$= e^{-\xi\omega_n\frac{\pi}{\omega_d}}\left(\cos\pi + \frac{\xi}{\sqrt{1-\xi^2}}\sin\pi\right) - 1$$

$$= e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi}$$

$$\boxed{\sigma_p = e^{-\frac{\sigma}{\omega_d}\pi} * 100 \quad [\%]}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\sigma = \zeta\omega_n$$

```
>> cos(pi)

ans =
```

$\boxed{}$

```
>> sin(pi)

ans =
```

$\boxed{}$

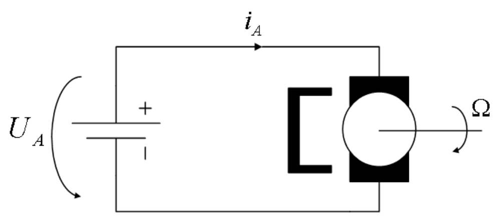
Determination of transient parameters

Settling time t_s :

$$t_s = 4T = \frac{4}{\sigma} = \boxed{}$$

$$t_s = 3T = \frac{3}{\sigma} = \boxed{}$$

DC motor model

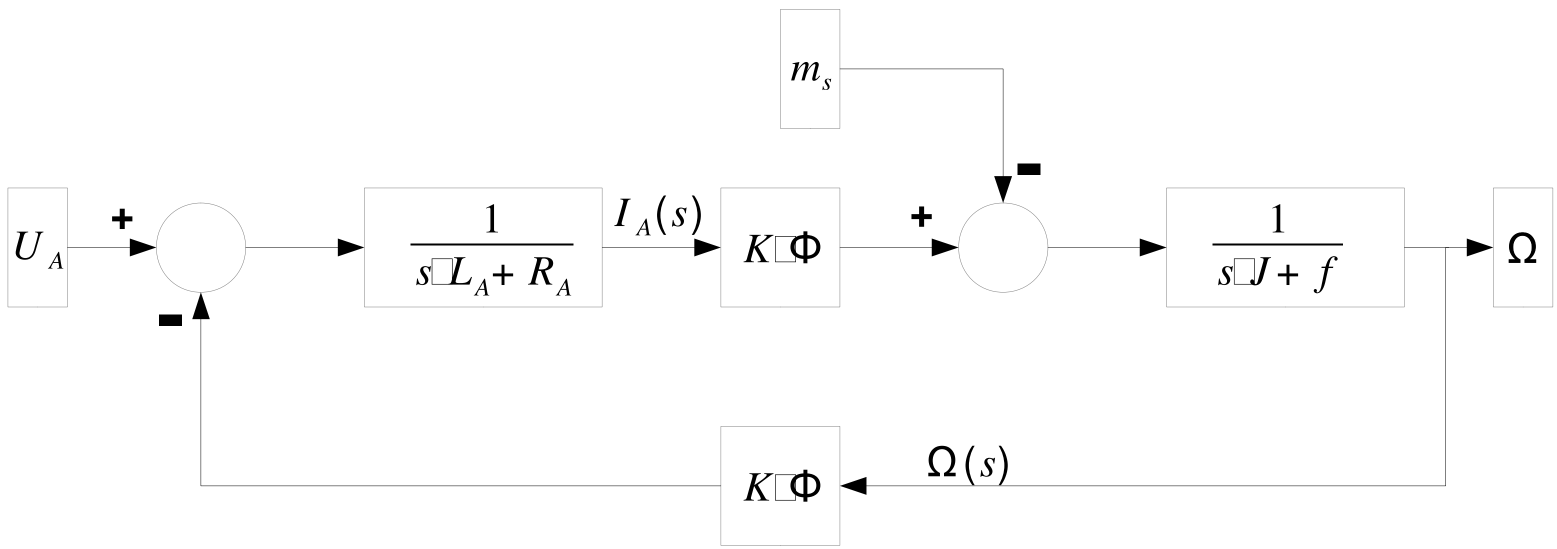


$$\begin{cases} U_A = R_A i_A(t) + L_A \dfrac{di_A(t)}{dt} + e(t) \\ J \dfrac{d\Omega(t)}{dt} = m - m_f - m_s \end{cases}$$

$$K\Phi_n =$$

DC motor model

$$\begin{cases} I_A(s) = \left[U_A - K \cdot \Phi \cdot \Omega(s)\right] \cdot \dfrac{1}{R_A + s \cdot L_A} \\ \Omega(s) = \left[K \cdot \Phi \cdot I_A(s) - m_s\right] \cdot \dfrac{1}{s \cdot J + f} \end{cases}$$



DC motor transfer function

$$\Omega(s) = \frac{K\Phi}{sJ + f} I_A(s) - \frac{m_s}{sJ + f} = \ldots$$

$$\ldots = \frac{K\Phi}{sJ + f} \frac{U_A(s) - K\Phi\Omega(s)}{R_A + sL_A} - \frac{m_s}{sJ + f} = \ldots$$

$$\ldots = \frac{K\Phi U_A(s) - (K\Phi)^2 \Omega(s)}{(sJ + f)(R_A + sL_A)} - \frac{m_s}{sJ + f}$$

$$\Omega(s) = \frac{K\Phi U_A(s) - (K\Phi)^2 \Omega(s)}{(sJ + f)(R_A + sL_A)}$$

$$\frac{\Omega(s)}{U_A(s)} = \frac{K\Phi}{(sJ + f)(R_A + sL_A) + (K\Phi)^2} = \ldots$$

$$\ldots = \frac{K\Phi}{s^2 L_A J + s(L_A f + R_A J) + R_A f + (K\Phi)^2} = \ldots$$

DC motor transfer function

$$\frac{\Omega(s)}{U_A(s)} = \frac{K\Phi}{(sJ+f)(R_A+sL_A)+(K\Phi)^2} = \dots$$

$$\dots = \frac{K\Phi}{s^2 L_A J + s(L_A f + R_A J) + R_A f + (K\Phi)^2} = \dots$$

$$\dots = \frac{\dfrac{K\Phi}{L_A J}}{s^2 + s\dfrac{L_A f + R_A J}{L_A J} + \dfrac{R_A f + (K\Phi)^2}{L_A J}} =$$

$$\frac{\dfrac{K\Phi}{L_A J}}{\sqrt{\dfrac{R_A f + (K\Phi)^2}{L_A J}}} \frac{\sqrt{\dfrac{R_A f + (K\Phi)^2}{L_A J}}}{s^2 + s\dfrac{L_A f + R_A J}{L_A J} + \dfrac{R_A f + (K\Phi)^2}{L_A J}}$$

$G(s) =$

$K_1 =$

$\omega_n =$

$2\zeta\omega_n$

DC motor dynamic parameters

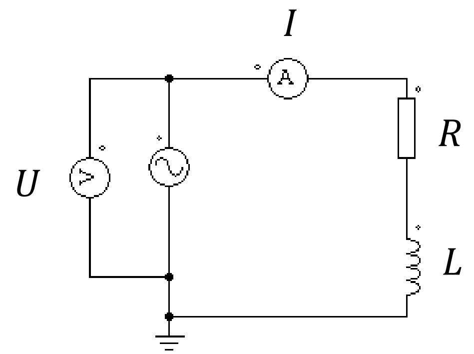
$$\omega_n =$$

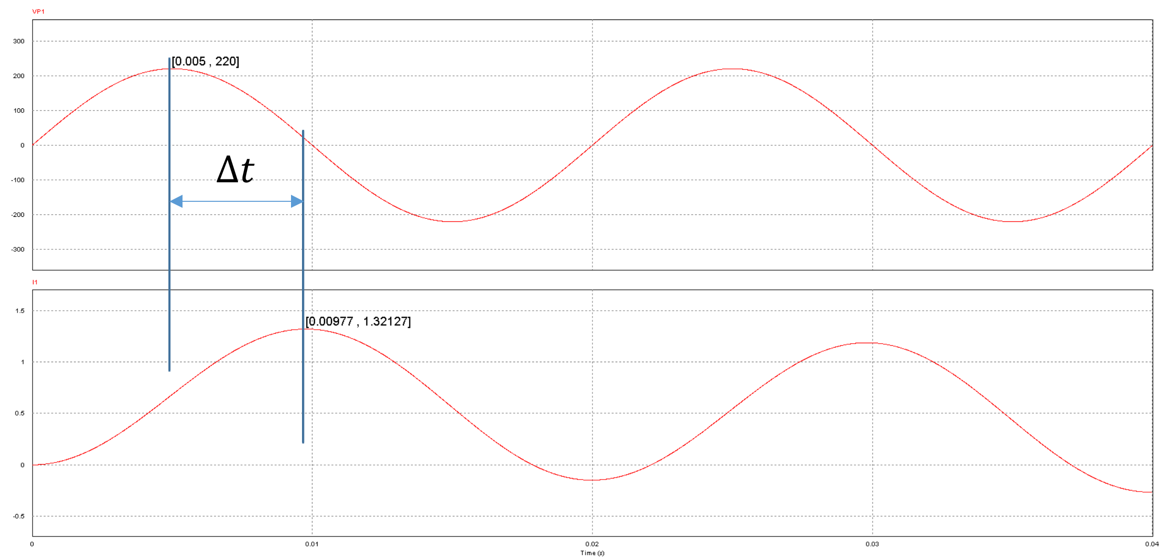
$$2\zeta\omega_n$$

$$\begin{cases} J = \dfrac{R_A f + (K\Phi)^2}{L_A \omega_n^2} \\ f = \dfrac{(2\xi\omega_n L_A - R_A)(K\Phi)^2}{L_A^2\omega_n^2 + R_A^2 - 2\xi\omega_n L_A R_A} \end{cases}$$

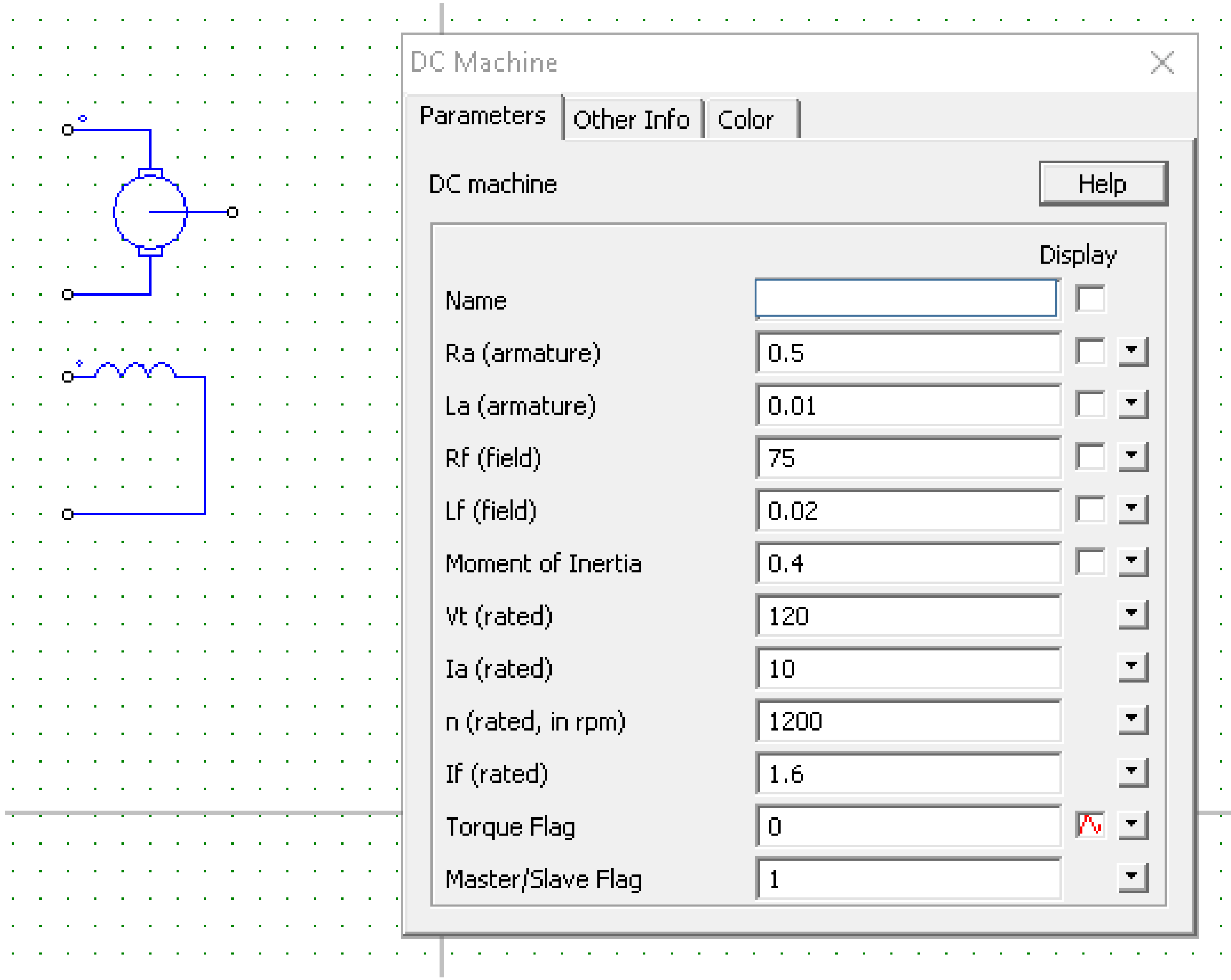
R-L circuit

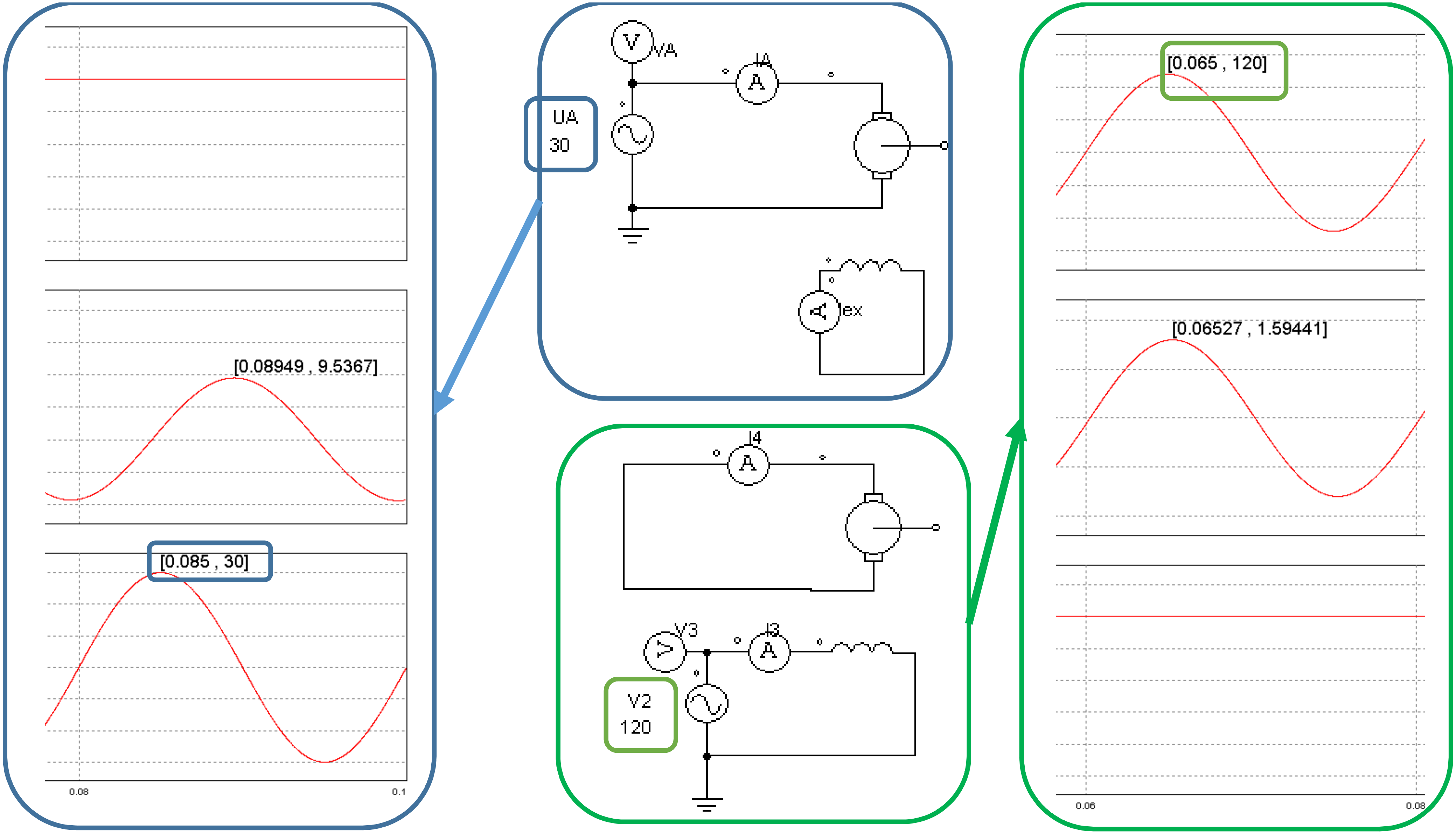
- $\omega = 2\pi f$
- $X_L = \omega L = R \operatorname{tg} \varphi$
- $U_{max} = I_{max}\sqrt{R^2 + {X_L}^2}$
- $\varphi = \omega \Delta t \frac{360^0}{2\pi}$
- Δt – voltage and current delay





Example:


DC Machine
Parameters
Other Info
Color
DC machine
Help
Display
Name
Ra (armature) 0.5
La (armature) 0.01
Rf (field) 75
Lf (field) 0.02
Moment of Inertia 0.4
Vt (rated) 120
Ia (rated) 10
n (rated, in rpm) 1200
If (rated) 1.6
Torque Flag 0
Master/Slave Flag 1



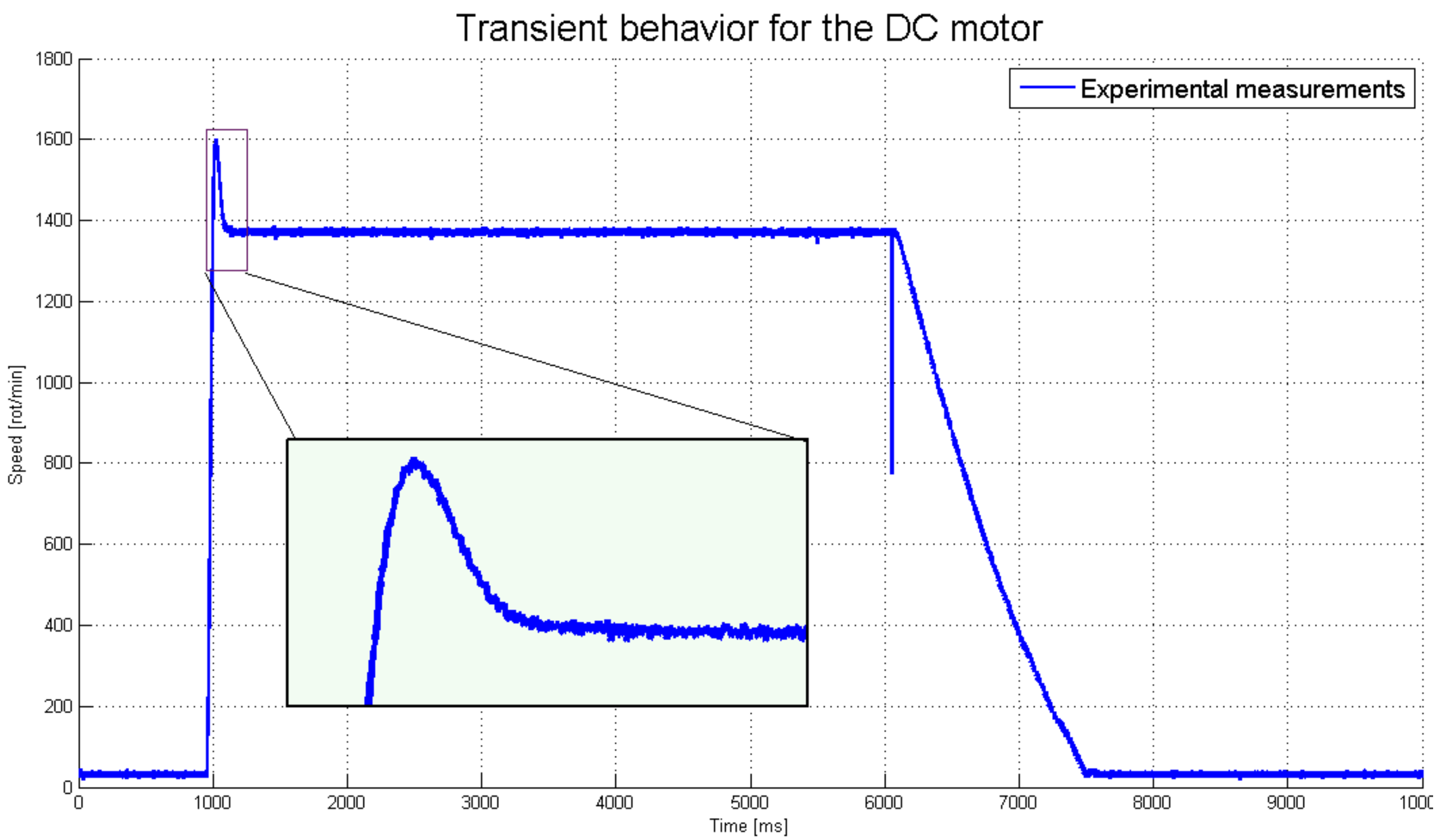
[0.08949 , 9.5367]
[0.085 , 30]
0.08
0.1
VA
IA
UA
30
Iex
I4
V3
I3
V2
120
[0.065 , 120]
[0.06527 , 1.59441]
0.06
0.08

DC motor nominal data

	DC nominal data		
	Data	Value	[]
1	U_A		V
2	I_A		A
3	n_n		rot/min

	Measured data		
	Data	Value	[]
1	R_A		Ω
2	L_A		H

Transient behavior of the DC motor



J and *f* determination

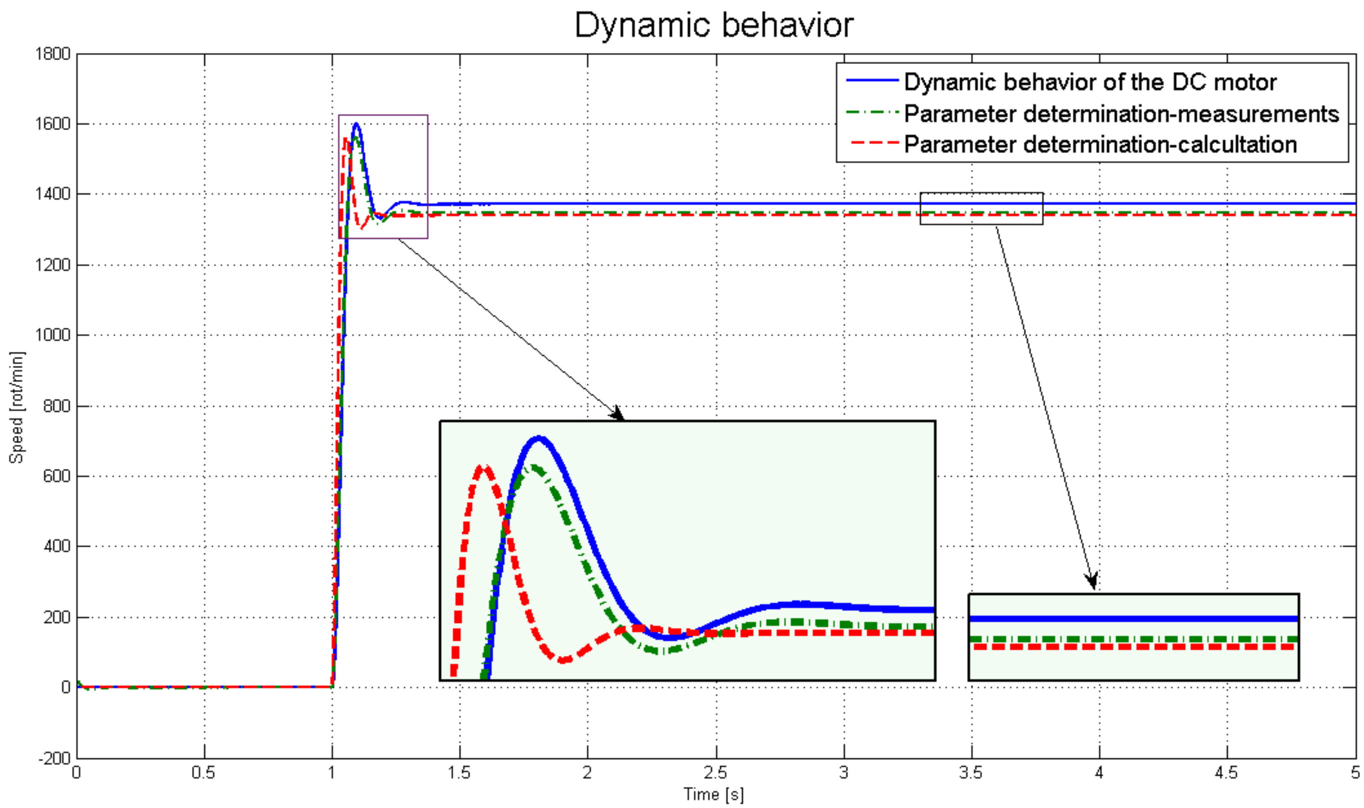
$t_p = 0{,}064$

$\sigma_p = 0{,}1674\ s$

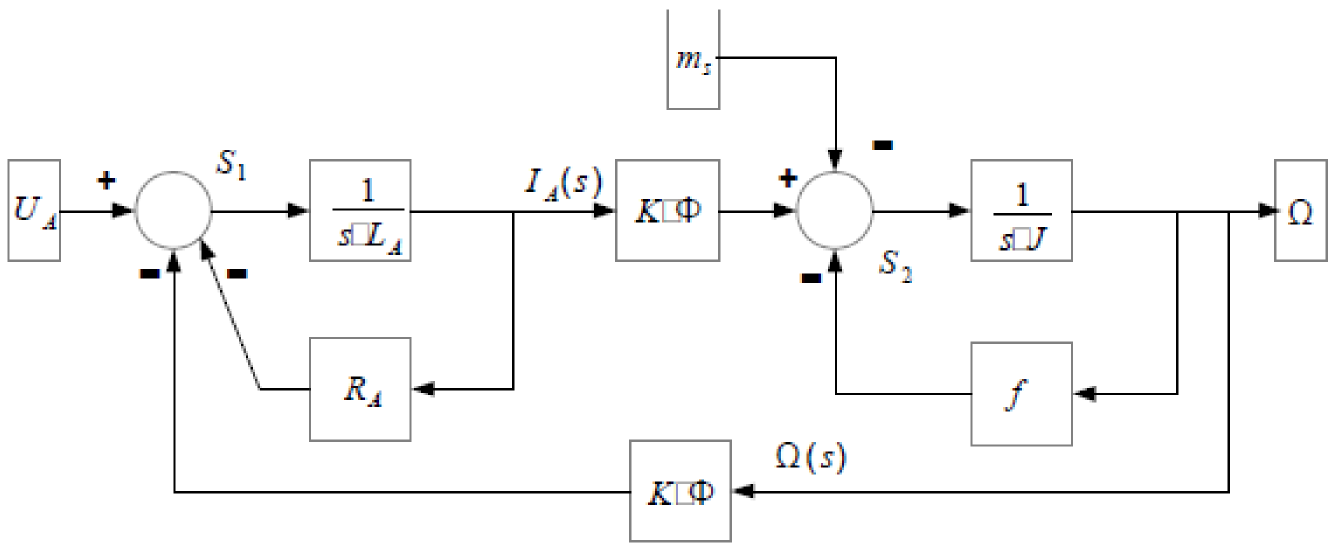
$$\begin{cases} \zeta = \sqrt{\dfrac{\left(\dfrac{\ln \sigma_p}{\pi}\right)^2}{1+\left(\dfrac{\ln \sigma_p}{\pi}\right)^2}} = \boxed{\quad} \\ \omega_n = \dfrac{\pi}{t_p\sqrt{1-\zeta^2}} = \boxed{\quad} \end{cases}$$

$$\begin{cases} J = \dfrac{R_A f + (K\Phi)^2}{L_A\,\omega_n^2} = \boxed{\quad} \\ f = \dfrac{(2\xi\omega_n L_A - R_A)(K\Phi)^2}{L_A^2\omega_n^2 + R_A^2 - 2\xi\omega_n L_A R_A} = \boxed{\quad} \end{cases}$$

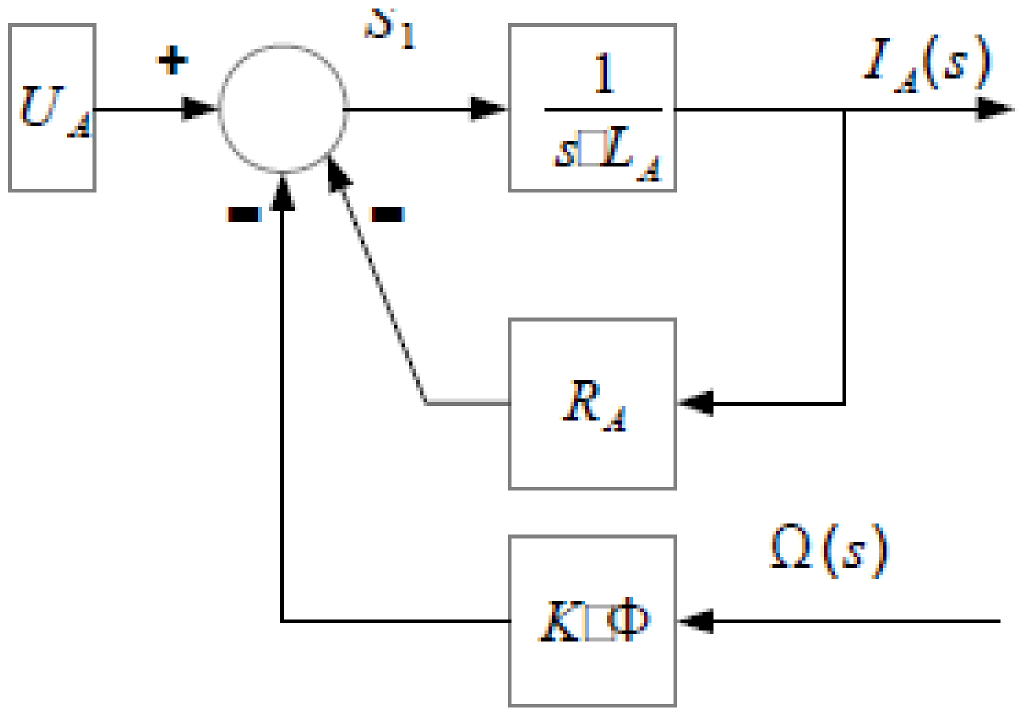
Experimental data



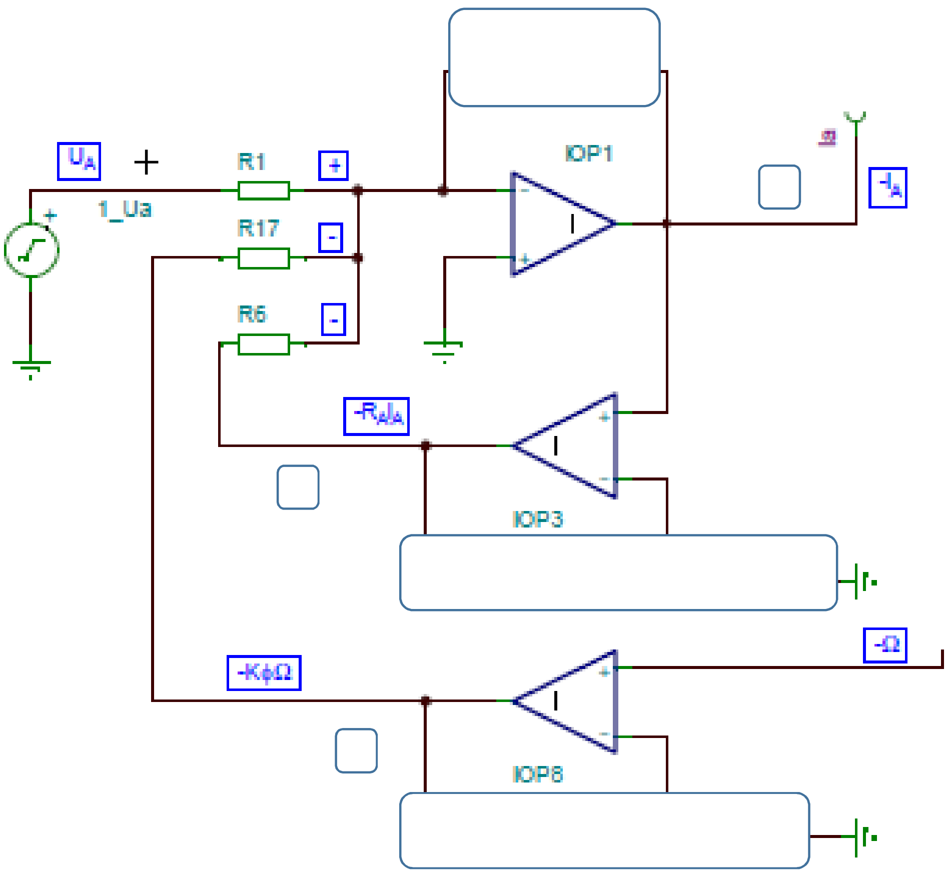
DC motor block schematic



DC motor block schematic - electric



DC motor equivalent schematic

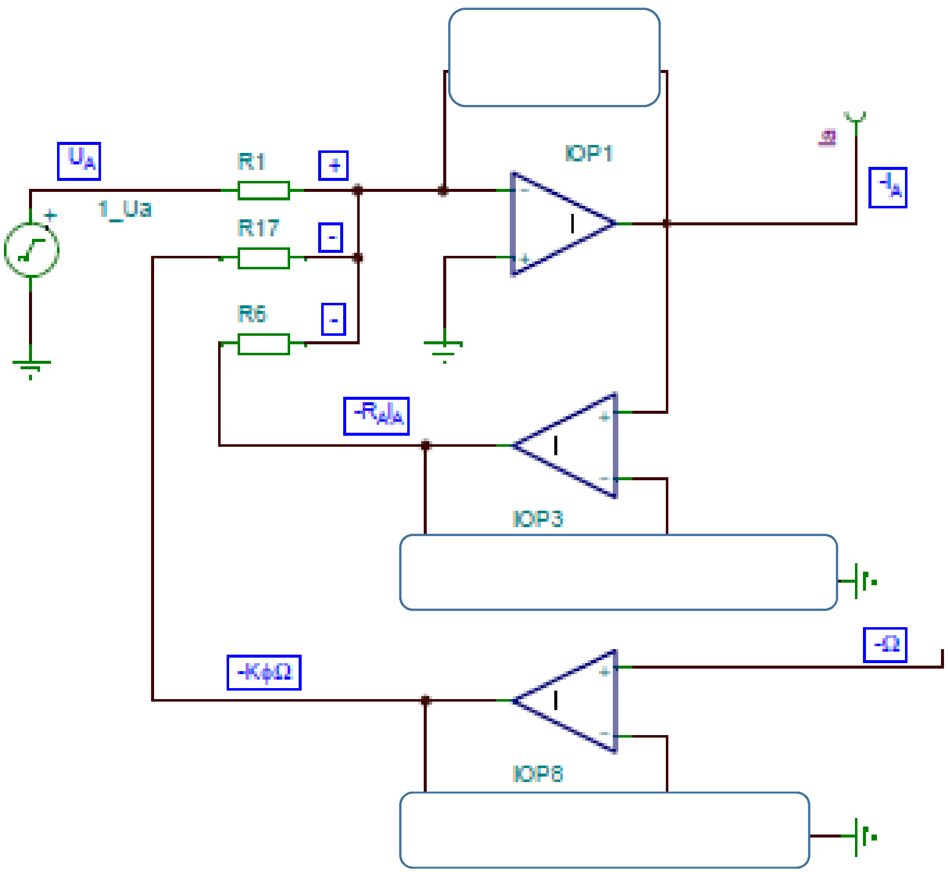


$$\begin{cases} V_{outIOP3} = \\ V_{inIOP3} = -I_A \\ V_{outIOP3} = -R_A \end{cases} \Rightarrow$$

$$\begin{cases} I_A(s) = \left[U_A - K\Phi V_{\Omega}(s) - R_A V_{I_A}(s)\right] \cdot \dfrac{1}{s \cdot L_A} \\ V_{outIO1} = -\dfrac{1}{s \cdot C_1 R_{el\_circuit}} \end{cases}$$

$$\Rightarrow [L_A] = C_1 R_{el\_circuit}$$

DC motor equivalent schematic

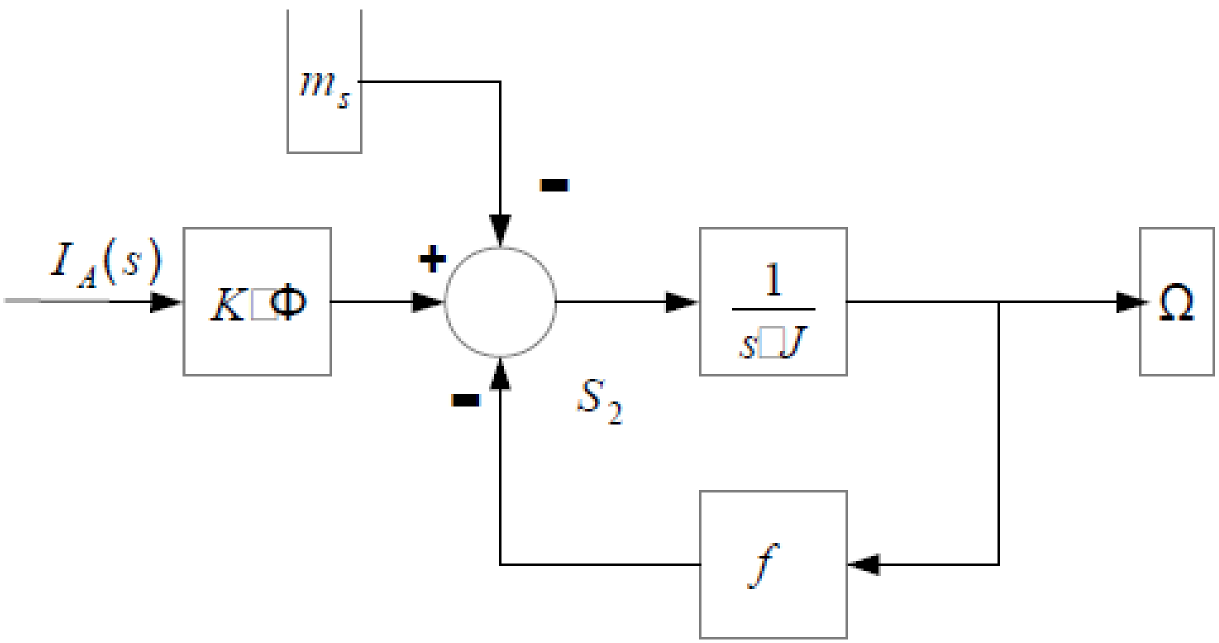


$$\begin{cases} V_{outIOP8} = \\ V_{inIOP8} = -V_{\Omega} \\ V_{outIOP8} = -[K\Phi]V_{\Omega} \end{cases} \Rightarrow$$

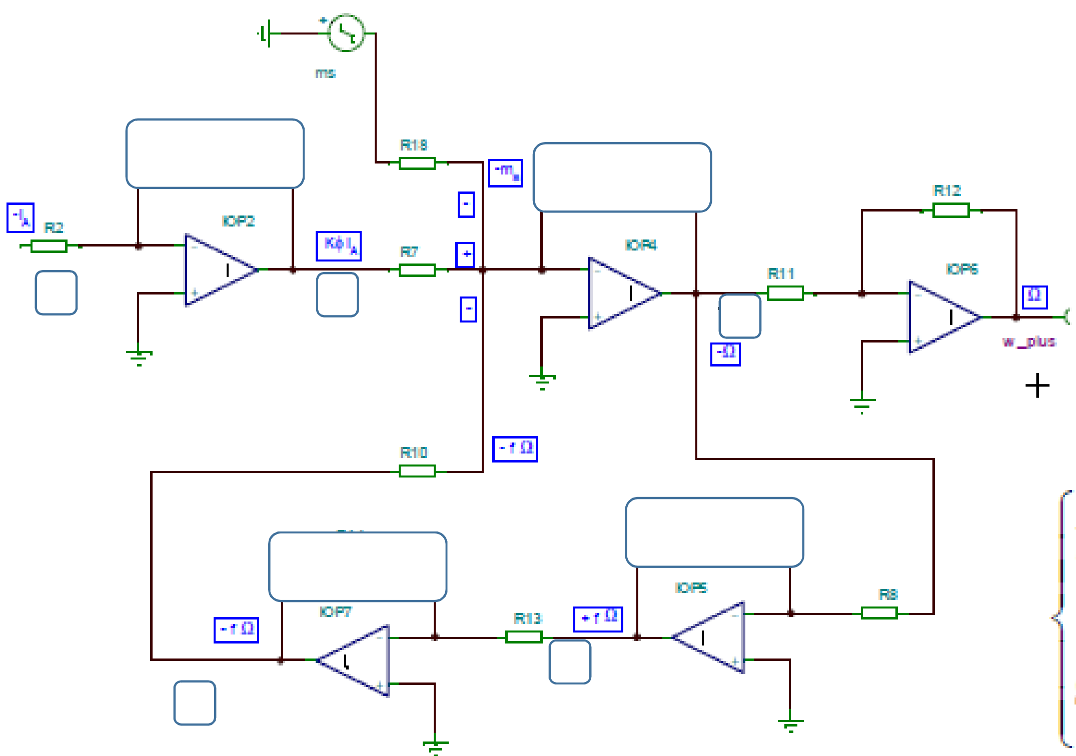
$$\begin{cases} V_{outIO1} = -\dfrac{1}{s \cdot C_1} \cdot \left(\dfrac{U_A}{R_1} - \dfrac{[K\Phi]V_{\Omega}}{R_{17}} - \dfrac{R_A V_{I_A}}{R_6} \right) \\ R_1 = R_{17} = R_6 = R_{el\_circuit} \end{cases}$$

$$V_{outIO1} = -\frac{1}{s \cdot C_1 R_{el\_circuit}} \cdot \left(U_A - [K\Phi]V_{\Omega} - R_A V_{I_A} \right)$$

DC motor block schematic - mechanic



DC motor equivalent schematic



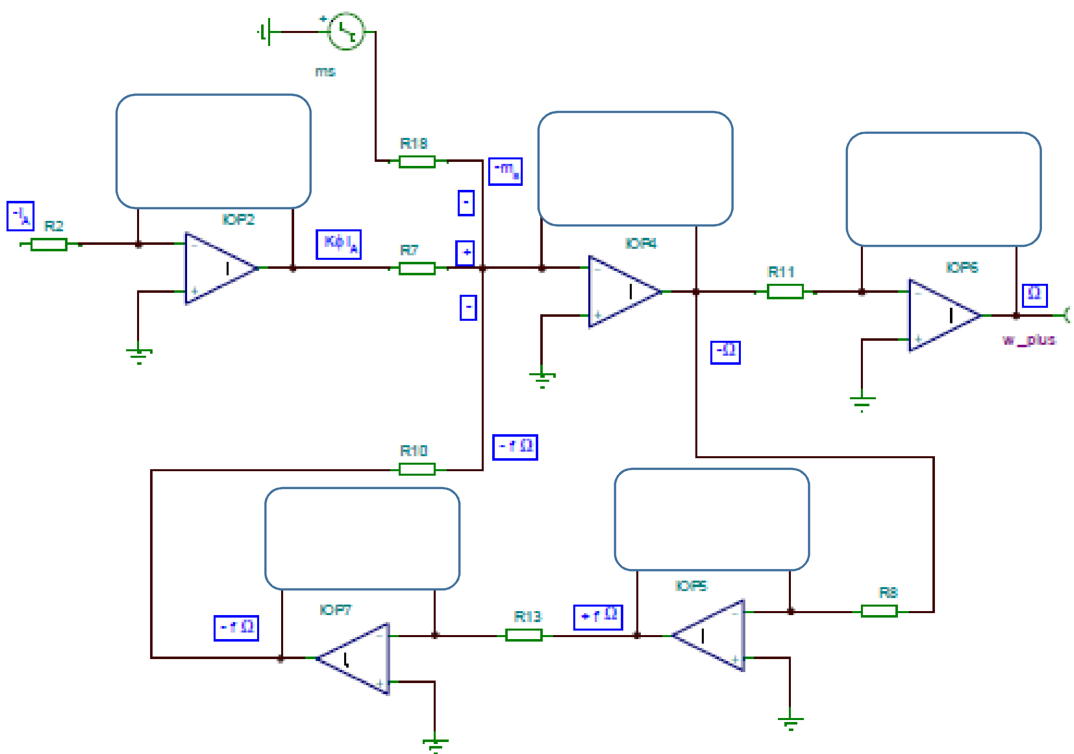
$$\begin{cases} V_{outIO4} = \\ R_7 = R_{10} = R_{mec\_circuit} \end{cases}$$

$$\Rightarrow \quad V_{outIO4} = -\frac{1}{s \cdot C_2 R_{mec\_circuit}} \left([K\Phi] V_{I_A} - [f] V_{\Omega} \right)$$

$$\begin{cases} V_{outIO4} = \\ \Omega(s) = \left[K\Phi V_{I_A}(s) - V_{m_s}(s) - [f] V_{\Omega}(s) \right] \cdot \frac{1}{s \cdot J} \end{cases}$$

$$\Rightarrow [J] = C_2 R_{mec \quad circuit}$$

DC motor equivalent schematic

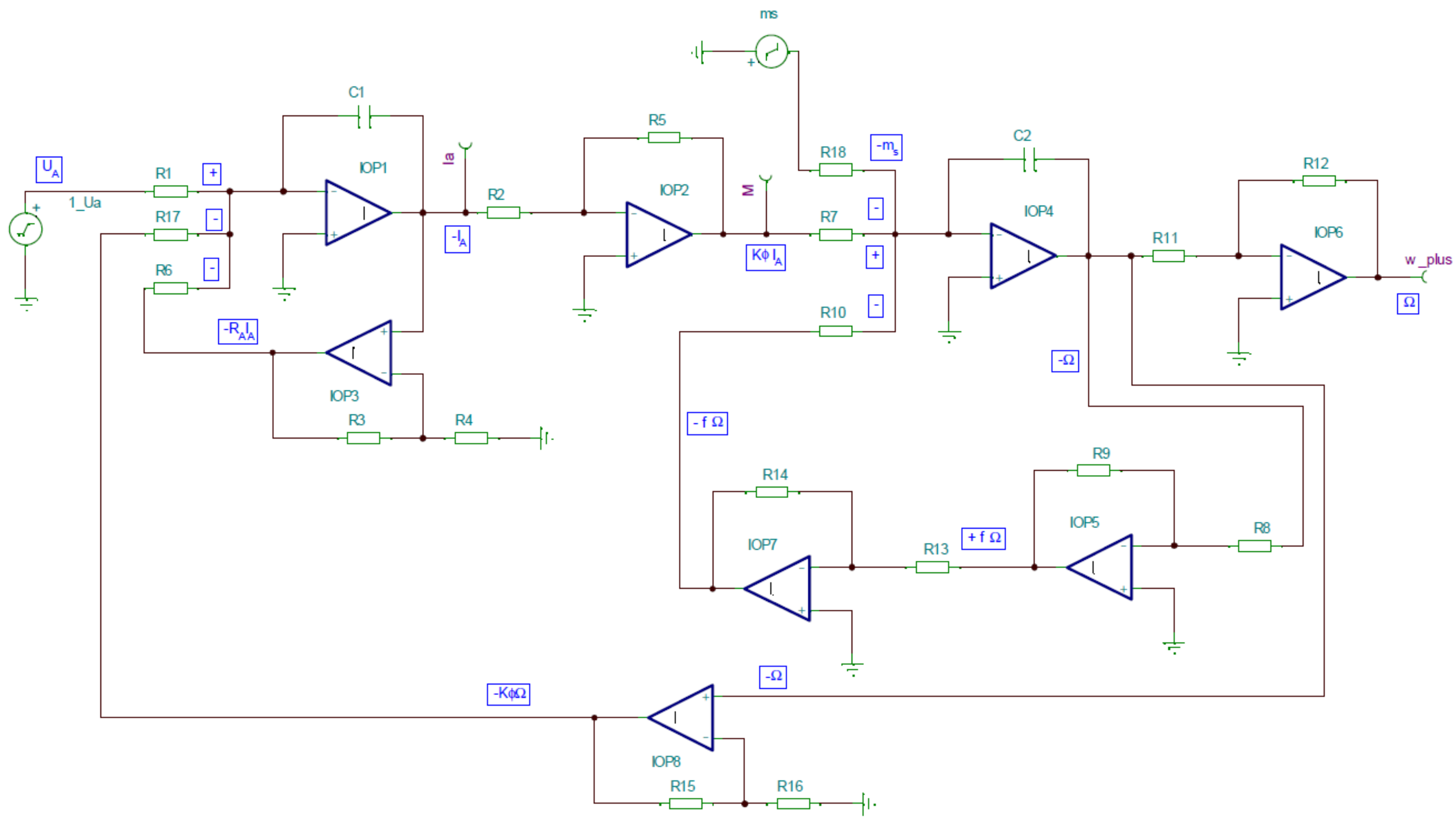


$$\begin{cases} V_{outIOP2} = \\ V_{inIOP2} = -V_{I_A} \\ V_{outIOP2} = [K\Phi]V_{I_A} \end{cases} \Rightarrow [K\Phi] = \frac{R_5}{R_2}$$

$$\begin{cases} V_{outIOP7} = \quad = -\dfrac{R_{14}}{R_{13}} \cdot V_{outIOP5} \\ V_{outIOP5} = \quad = -\dfrac{R_9}{R_8} \cdot V_{outIOP4} \end{cases}$$

$$\begin{cases} V_{outIOP7} = \dfrac{R_{14}}{R_{13}} \dfrac{R_9}{R_8} \cdot V_{outIOP4} \\ V_{outIOP7} = -[f]V_{\Omega} \\ V_{outIOP4} = -V_{\Omega} \end{cases} \qquad [f] = \frac{R_{14}}{R_{13}} \frac{R_9}{R_8}$$

DC motor equivalent schematic

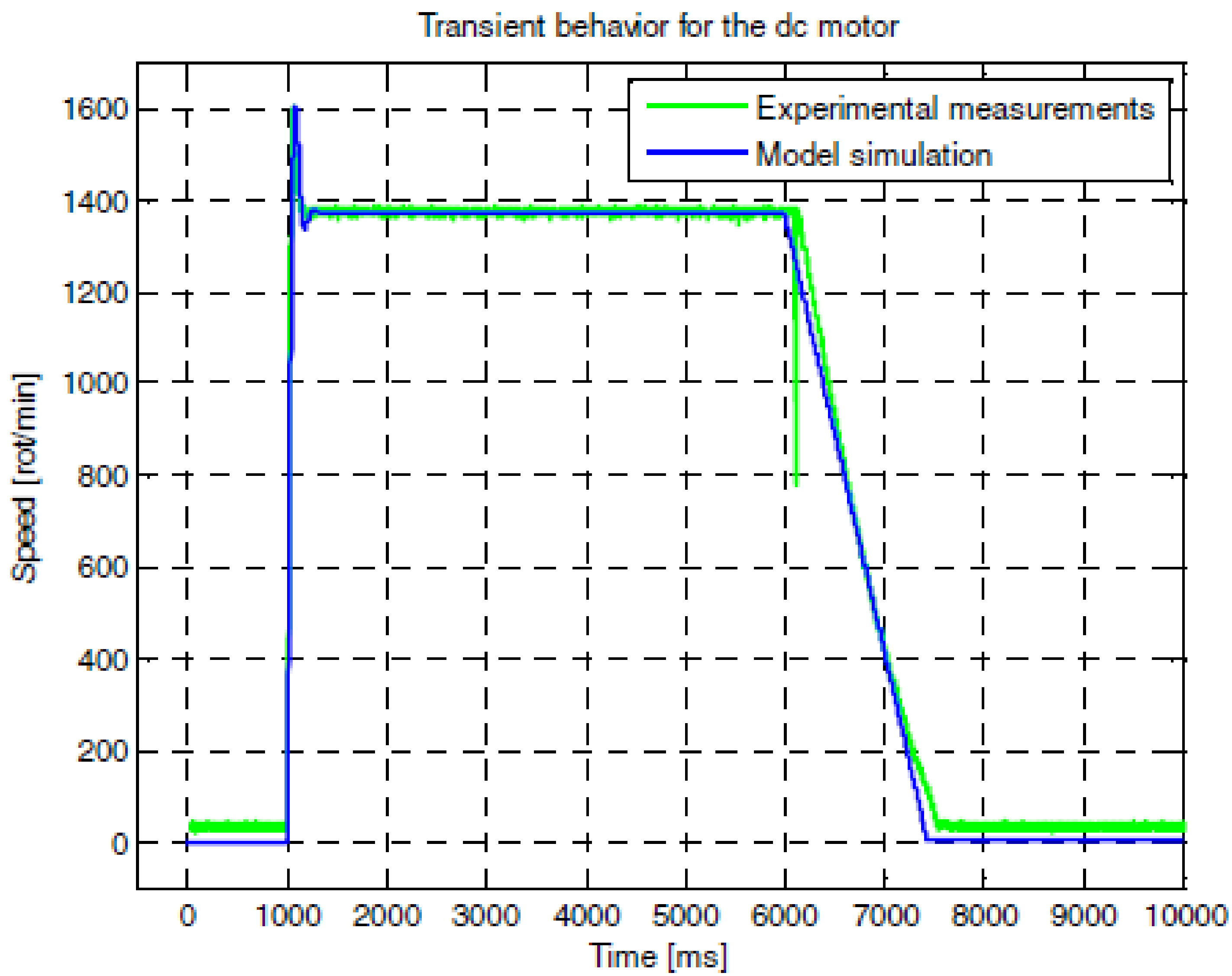


Experimental data

	Nominal data		
	Data	***Value***	***Unit***
1	UA		V
2	IA		A
3	RA		Ω
4	LA		H
5	nn		rot/min

	Measured data		
	Data	***Value***	***Unit***
1	J		kg·m2
2	f		Nm/rad/s

Experimental data



DC motor transient behavior