



Photovoltaic Modelling Introduction

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The photovoltaic effect

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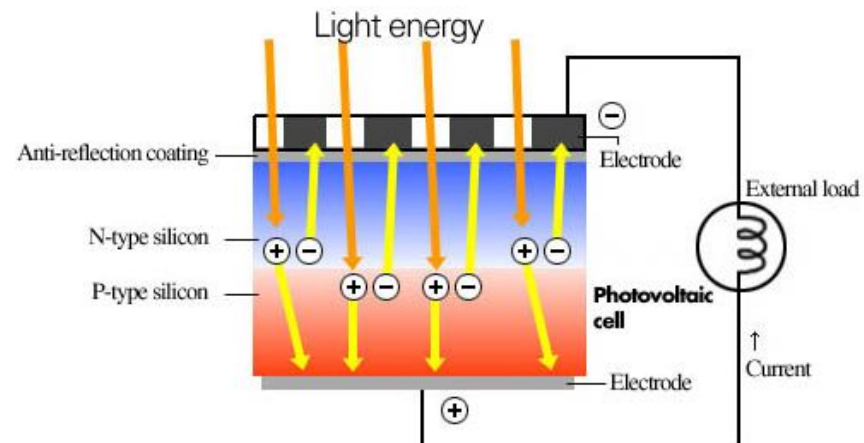
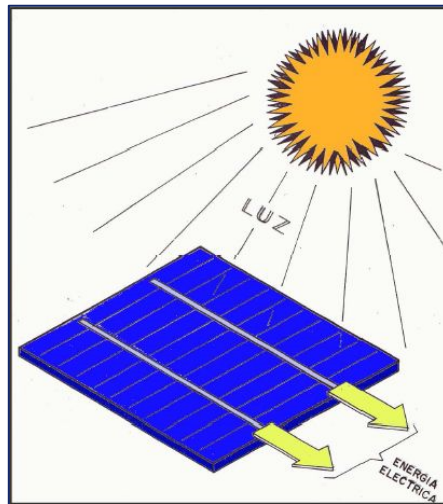
Photovoltaic Panel

3

Photovoltaic System

Photovoltaic effect

☞ *Sunlight striking a semiconductor and causing electrons to be excited due to energy in the sunlight (photons).*





Photovoltaic effect

Several important magnitudes must be used:

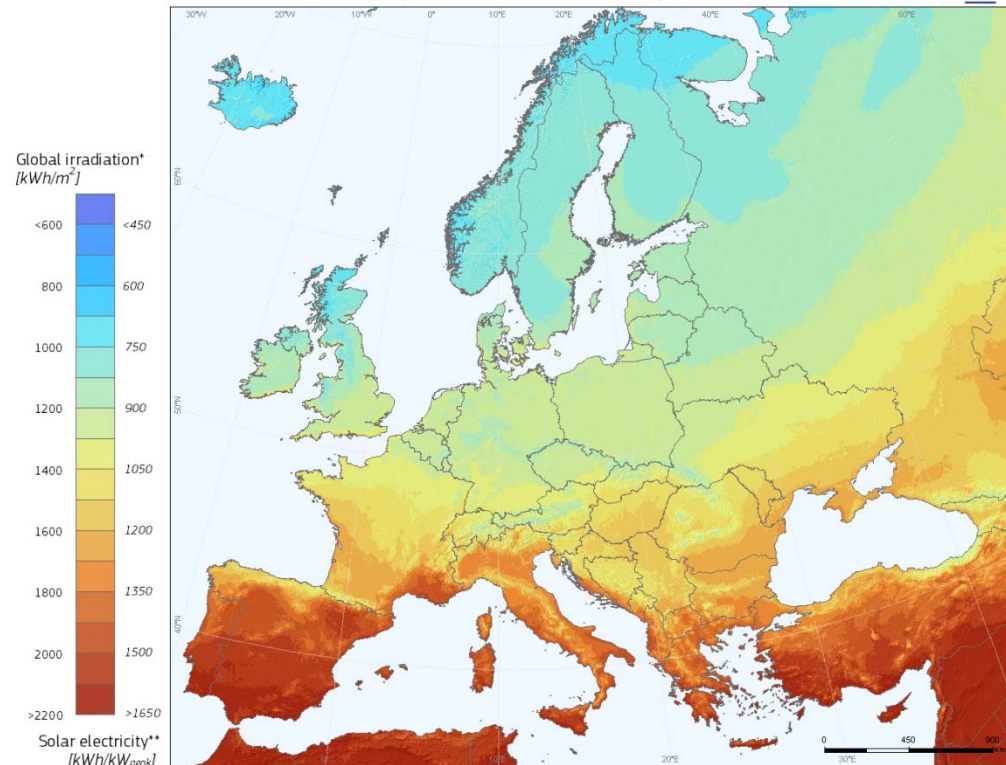
☞ *Irradiance*

☞ *Temperature*

☞ *Current*

☞ *Voltage*

Photovoltaic Solar Electricity Potential in European Countries



* Yearly sum of global irradiation incident on optimally-inclined south-oriented photovoltaic modules

** Yearly sum of solar electricity generated by optimally-inclined 1kW_p system with a performance ratio of 0.75

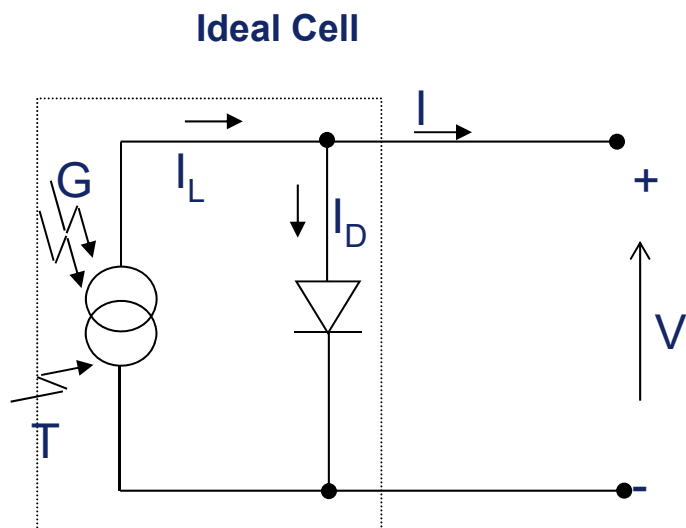
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PVGIS <http://re.jrc.ec.europa.eu/pvgis/>

Authors: Thomas Huld, Irene Pinedo-Pascua
EC - Joint Research Centre
In collaboration with: CM SAF, www.cmsaf.eu

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Photovoltaic effect

Ideal Cell



$$I = I_L - I_D \quad (1)$$

 Total current in the diode

$$I_D = I_0 \left[e^{\frac{qV}{aKT_C}} - 1 \right] \quad (2)$$

 The Current I_0 : appears with reverse polarity and relatively high negative tensions.

$$I = I_L - I_0 \left[e^{\frac{Vq}{aKT_C}} - 1 \right] \quad (3)$$



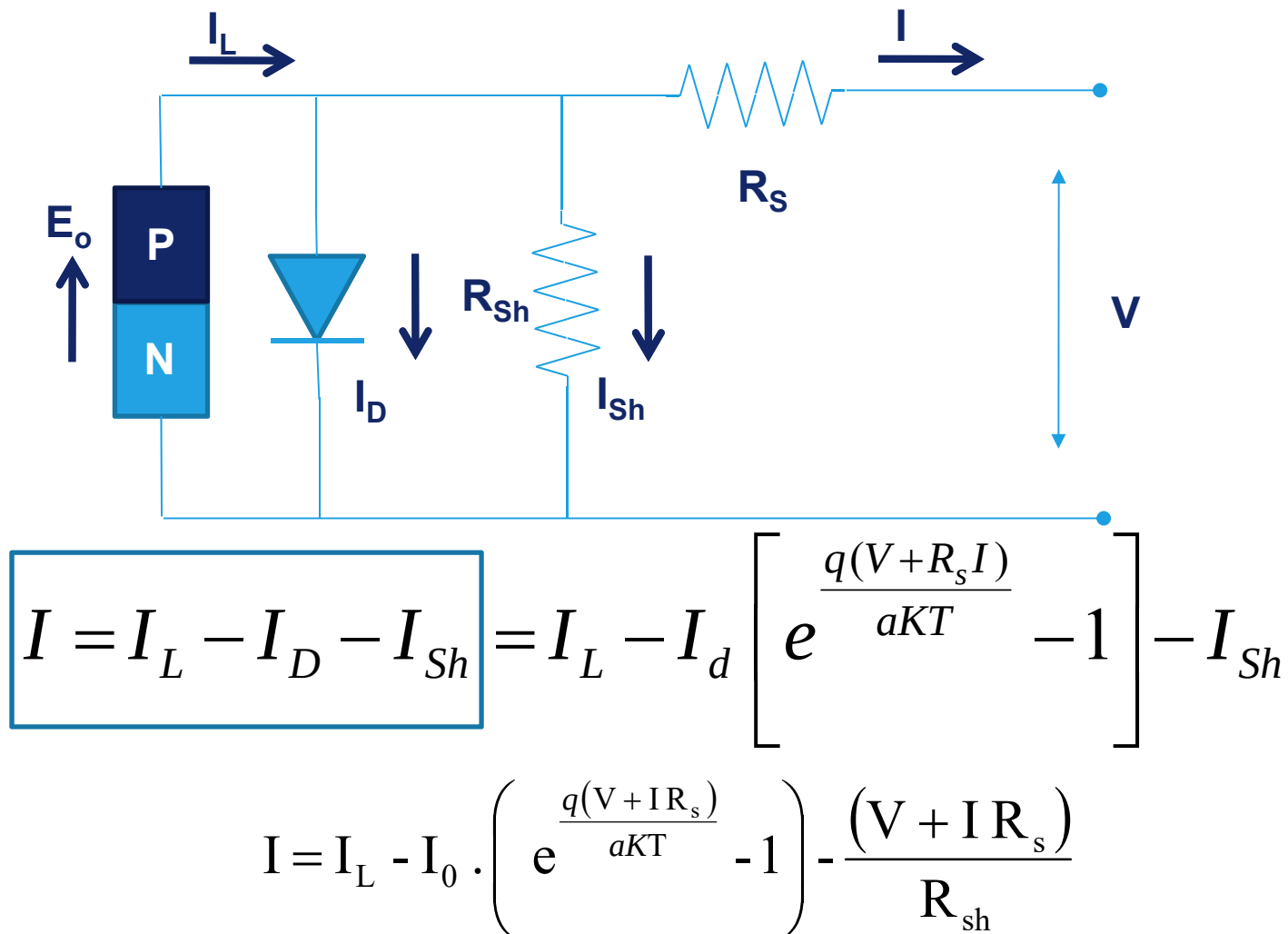
Other characteristics or data

$$I = I_L - I_0 \left[e^{\frac{Vq}{aKT_C}} - 1 \right]$$

- I_D : Diode current (A)
- I_0 : Saturation current of the diode (A)
- q : Electron charge, $1.6 \cdot 10^{-19}$ (C)
- a : Diode ideality factor.
- K : Boltzmann's Constant, $1.38 \cdot 10^{-23}$ J/K
- T_C : Cell Temperature ($^{\circ}\text{K}$)
- V : Voltage on the photovoltaic module (V)
- I : Current photovoltaic module (A)

Photovoltaic effect

Equivalent circuit of a photovoltaic cell





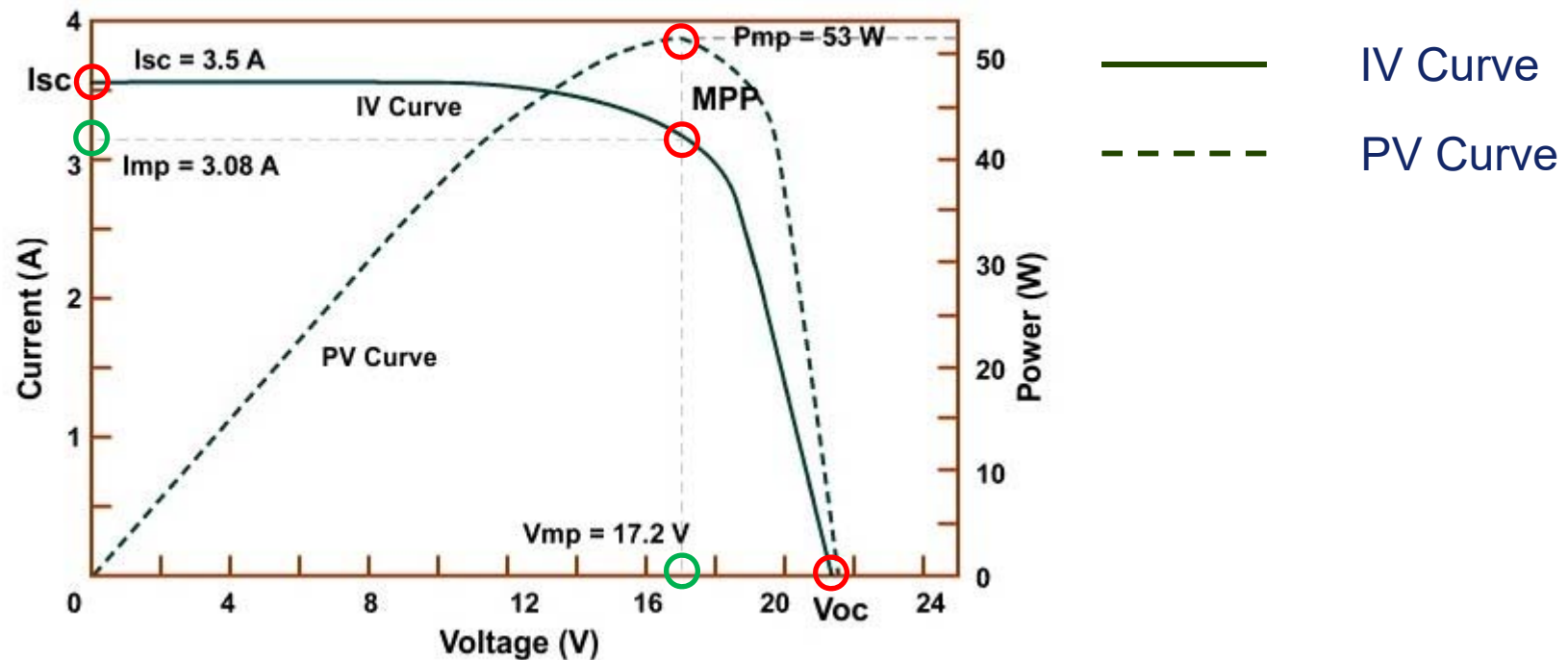
Photovoltaic effect

The fundamental electrical parameters:

- **Short circuit current (I_{sc})** is the current through the solar cell when the voltage across the solar cell is zero
- **Open circuit voltage (V_{oc})** is the difference of electrical potential between two terminals of a device when disconnected from any circuit. There is no external load
- **Maximum current (I_{max})** current at the maximum power point
- **Maximum voltage (V_{max})** current at the maximum power point
- **Maximum power (P_{max})**

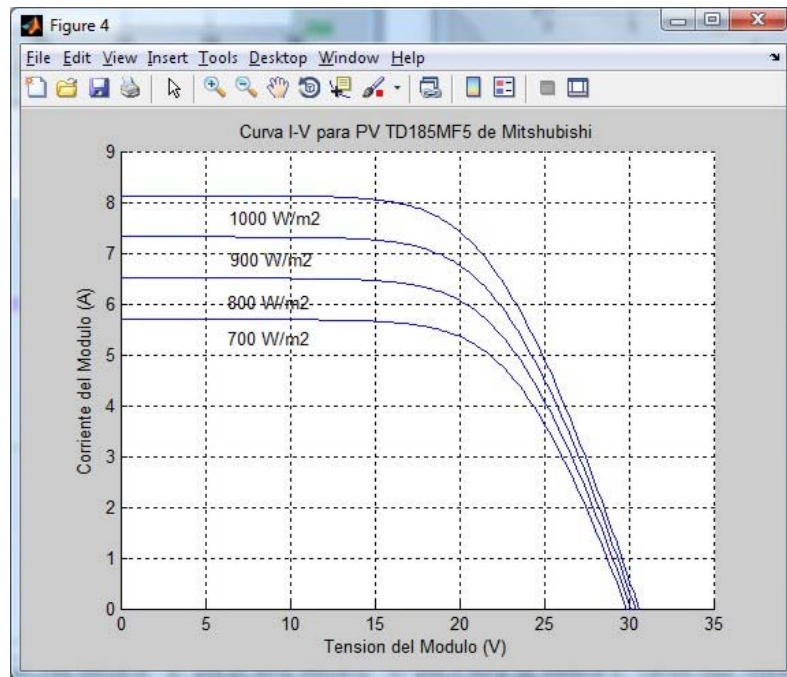
Photovoltaic effect

Characteristic curves (IV) and (PV)

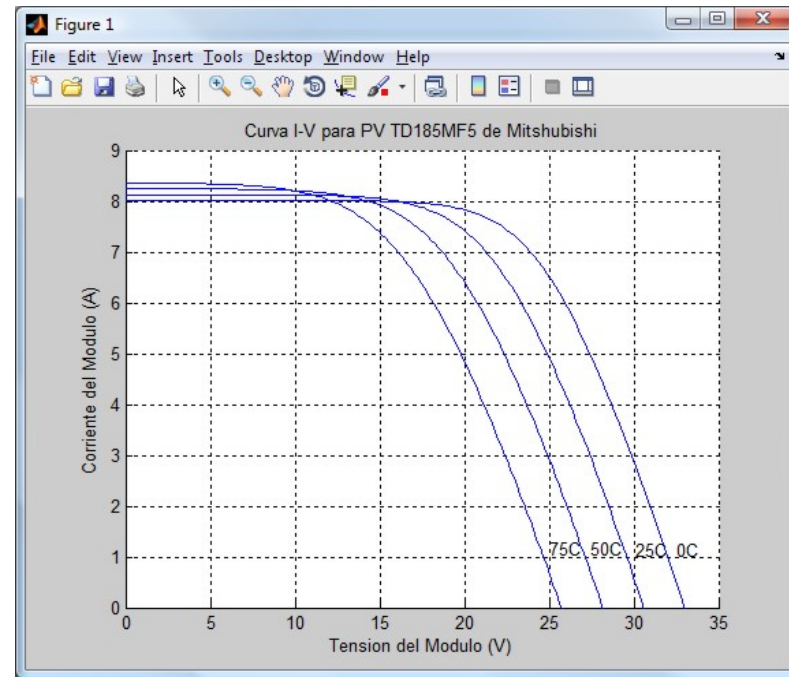


Photovoltaic effect

Types of IV characteristic curves:



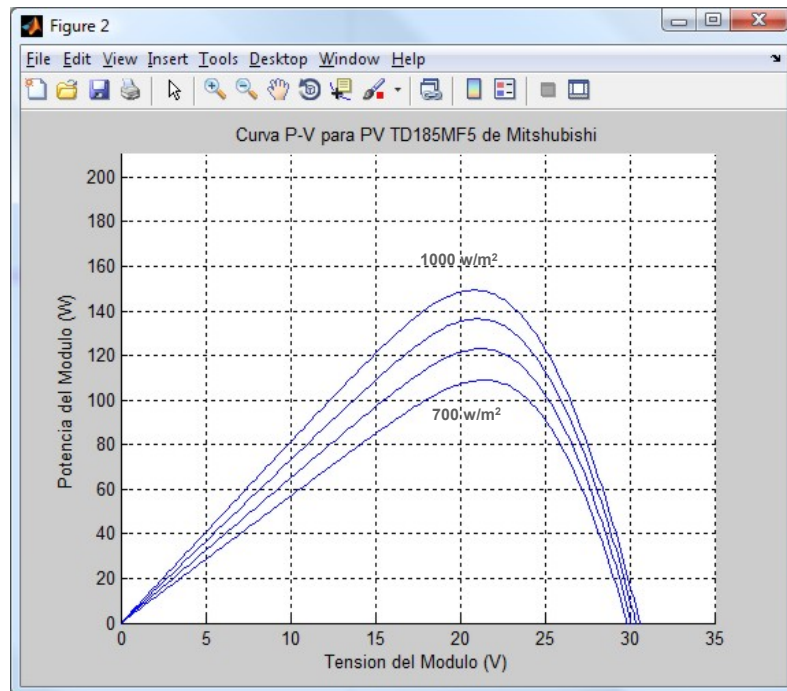
IV curve for constant T^a 25°C



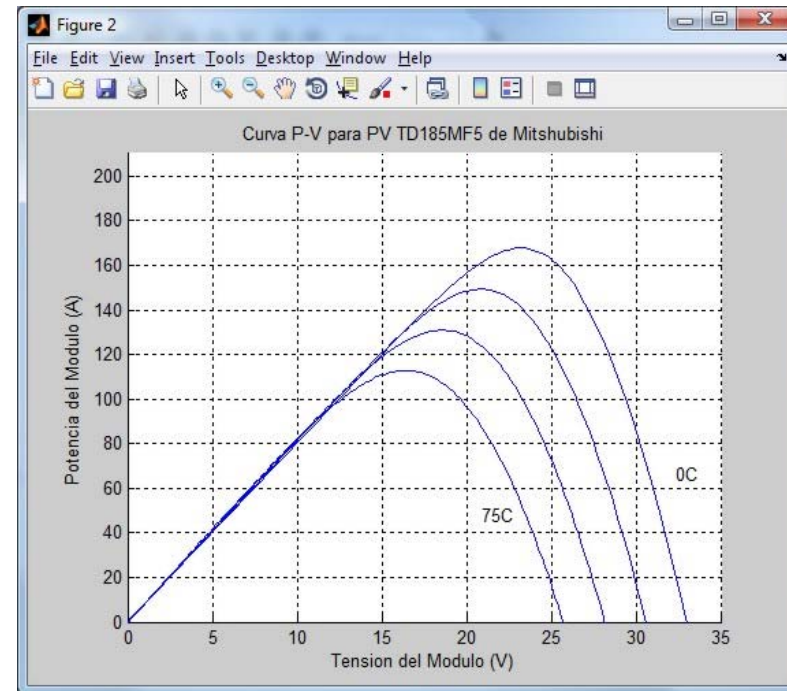
IV curve for constant G (Irradiance)
1000 W/m²

Photovoltaic effect

Types of PV characteristic curves:



PV curve for constant T^a 25°C

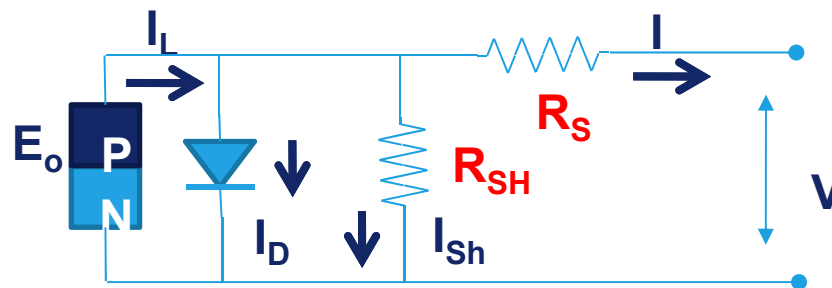


PV curve for constant G (Irradiance)
1000 W/m²

Photovoltaic effect

Series and shunt resistances:

- ☞ R_s : A function of the impurities and contact resistance.
- ☞ R_{sh} : Inversely proportional to the leakage current to earth.
- ☞ **Ideal photovoltaic cell**: $R_s = 0$ and $R_{sh} = \infty$.
- ☞ **Silicon cells**: $R_s = 0,05\text{--}0,1\Omega$ and $R_{sh} = 200\text{--}300\Omega \Rightarrow I_{sh} \approx 0$.
- ☞ The PV conversion efficiency is very sensitive to small variations in R_s , but insensitive to variations in R_{sh}

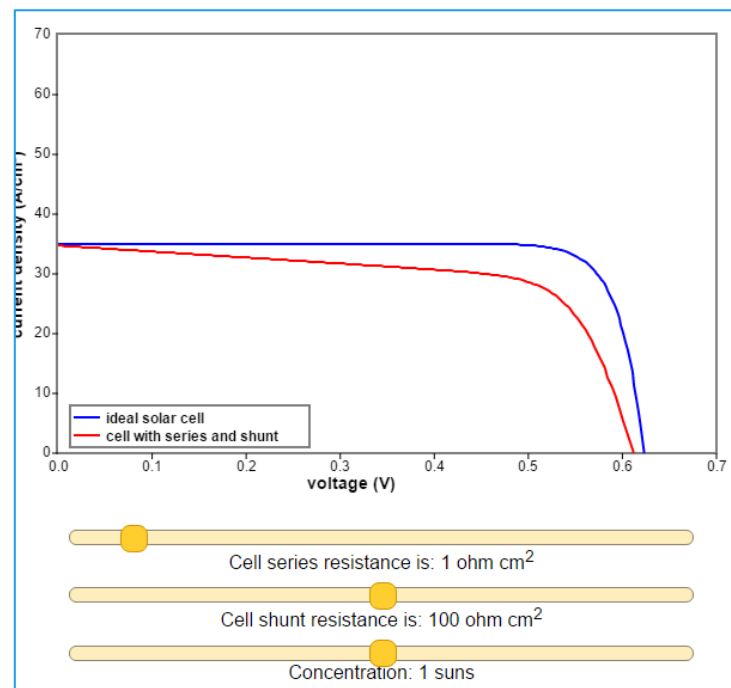




Photovoltaic effect

Series and shunt resistances:

☞ Influence of the R_S and R_{SH} on the curve characteristics



❖ Measurement conditions of a photovoltaic cell

■ Standard Test Conditions, STC

- Solar irradiance : 1 kW/m^2 .
- Air mass : PM 1,5.
- Cell temperature : 25°C .

The **air mass coefficient** defines the direct optical path length through the Earth's atmosphere, expressed as a ratio relative to the path length vertically upwards, i.e. at the zenith.

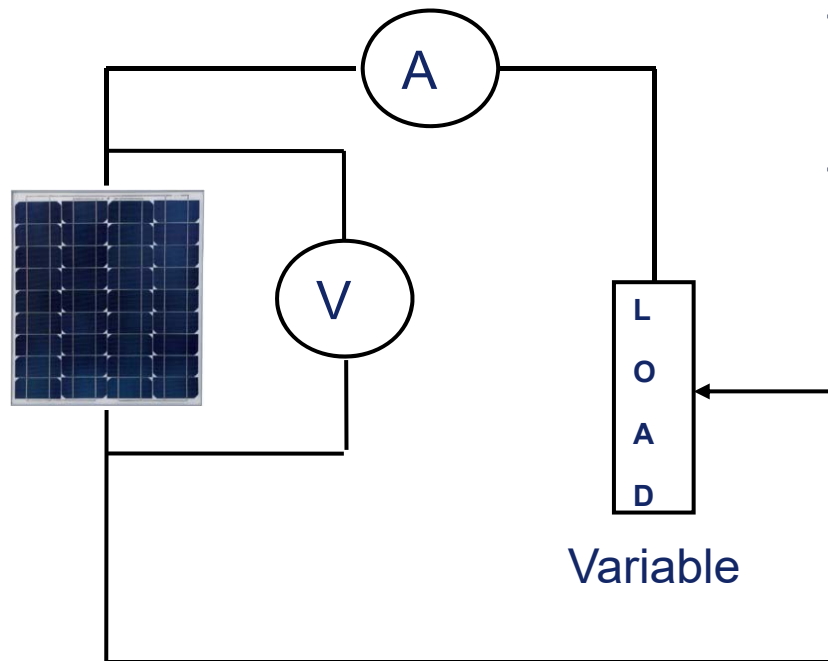
■ Condiciones de operación estándar, SOC

- Solar irradiance : $0,8 \text{ kW/m}^2$.
- Wind speed : 1 m/s .
- Environment temperature : 20°C .

■ Nominal operating temperature of cells, NOCT. (Open circuit).

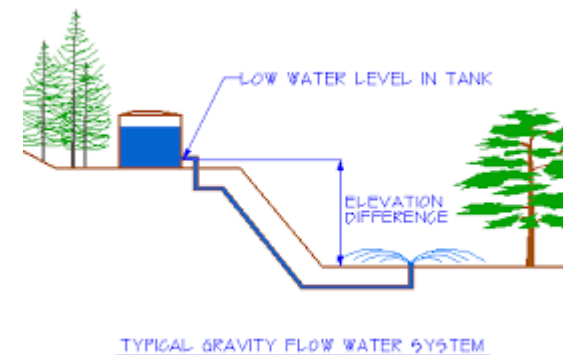
Photovoltaic effect

Data gathering for IV or PV curve building



The current is similar to measure flow of a river

The voltage is similar to measure slope of a river



Photovoltaic effect

Data gathering for IV or PV curve building

The elements used for measurements are:

☞ Sineax CAM



☞ TV809



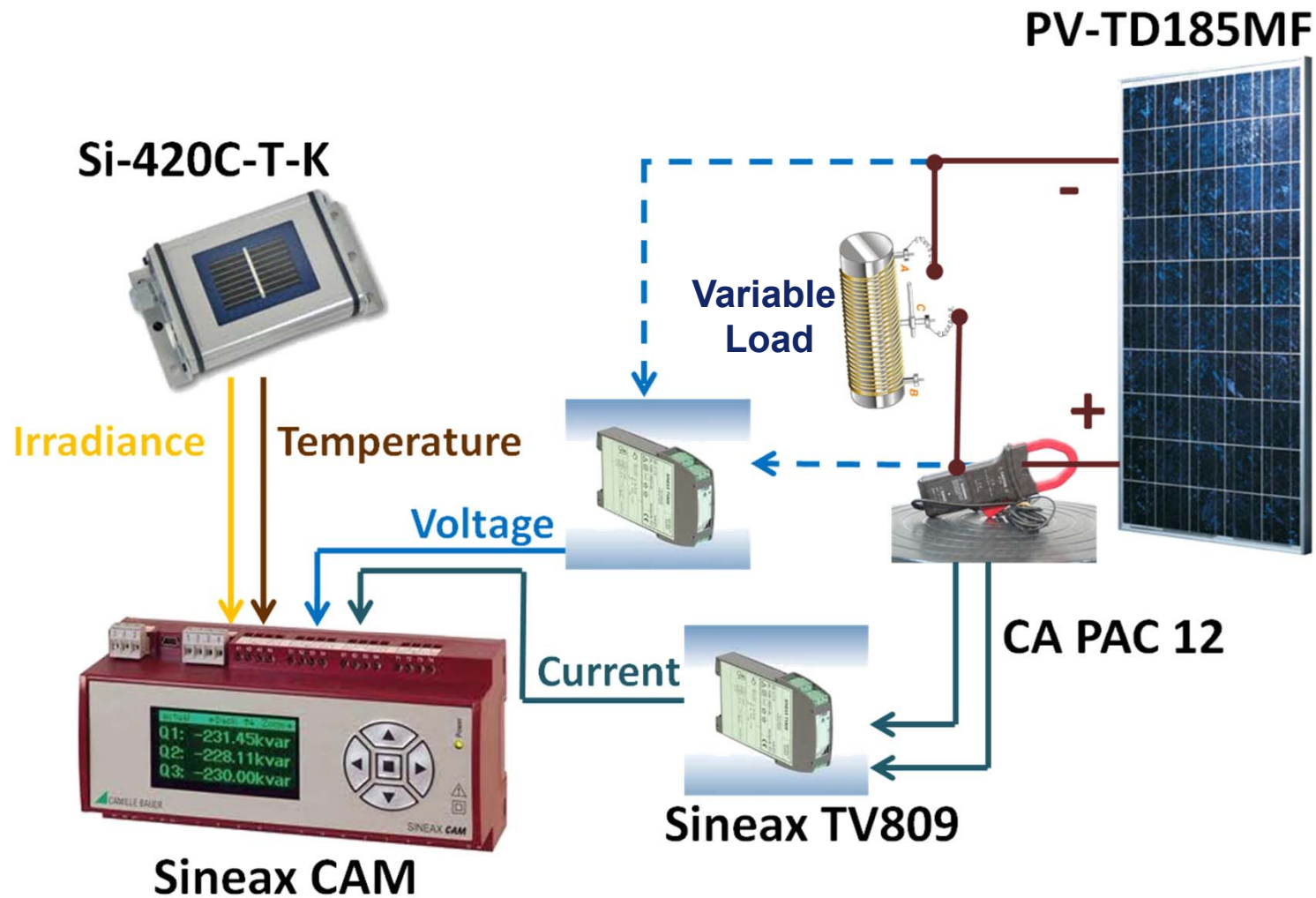
☞ Irradiance sensor (Si-420TC-T-K)



☞ Current clamps: Chauvin Arnoux PAC12: The PAC12

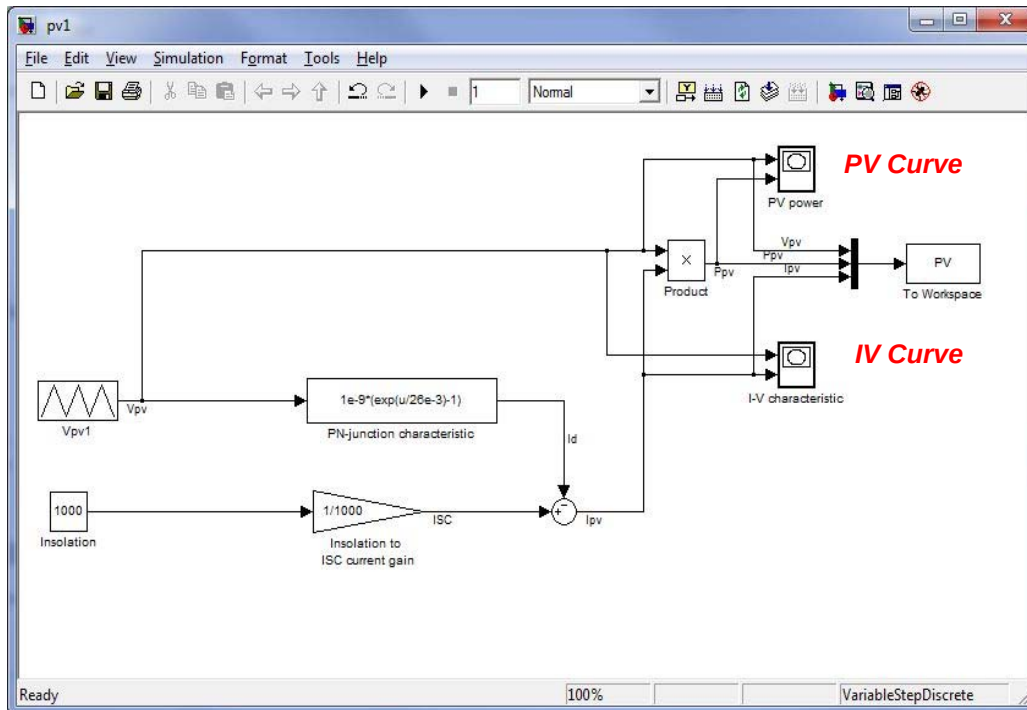


Photovoltaic effect

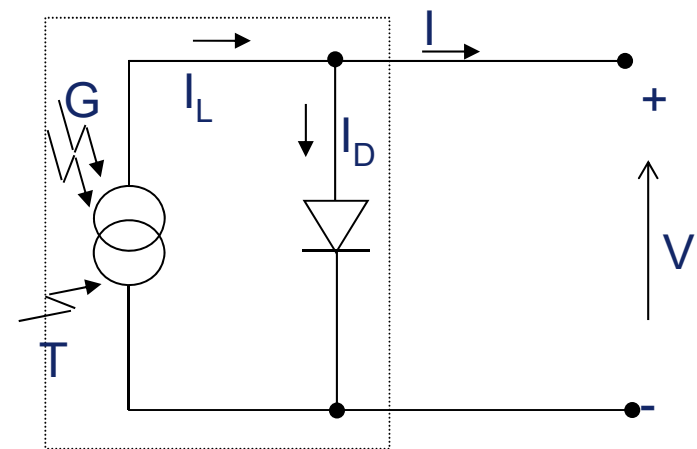


Photovoltaic effect

Mathematical Models of the photovoltaic cell:



Ideal Cell



$$I_{PV} = I_L - I_0 \left[e^{\frac{Vq}{aKT_C}} - 1 \right]$$



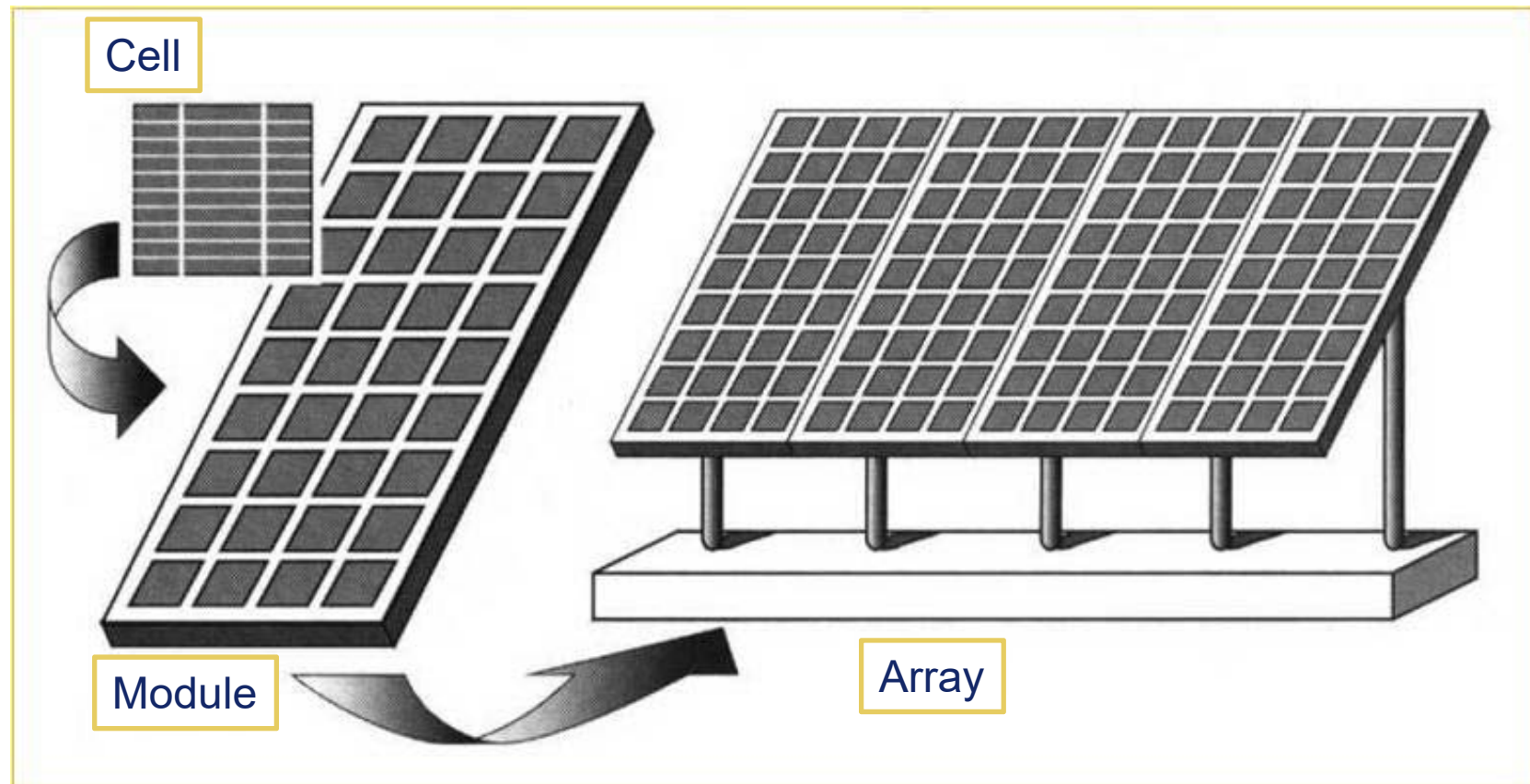
Basic Models

Complex Models

**Photovoltaic
Panel**



Photovoltaic Panel





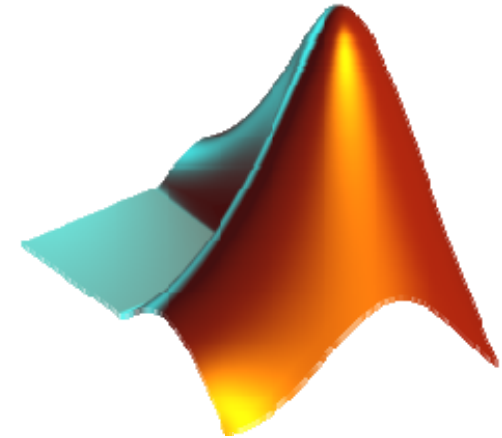
Key specifications of the PV Panel

- ☞ I_{sc} : Short Circuit Current
- ☞ V_{oc} : Open Circuit Voltage
- ☞ N_s : Number of cells in series
- ☞ P_{max} : Maximum power
- ☞ I_{mp} : Maximum power current
- ☞ V_{mp} : Maximum power voltage
- ☞ Normal operating cell temperature (NOCT)
- ☞ $\alpha_{I_{sc}}$: Temperature coefficient of I_{sc}
- ☞ $\beta_{V_{oc}}$: Temperature coefficient of V_{oc}



Basic Models

- **Matlab Tools**
- **Basic Model in Simulink**
- **Simulink model with Tags**
- **Physical component model**
- **Model advanced component library**
- **Model with real data**



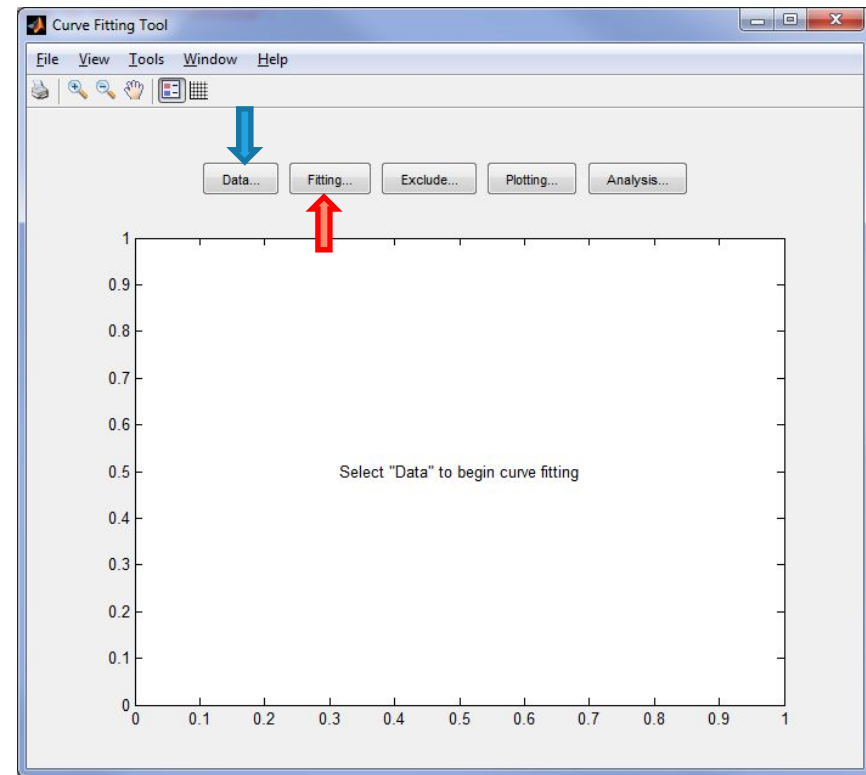
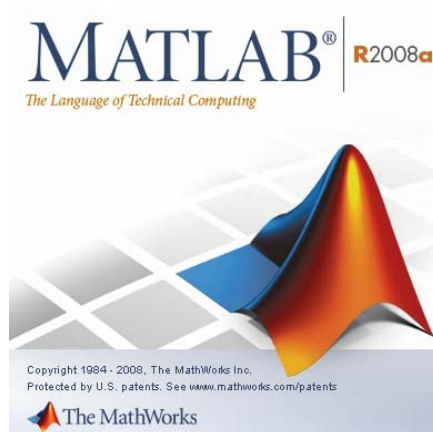


Matlab Tools

Curve Fitting Toolbox

>>cftool

>>cftool(V,I)

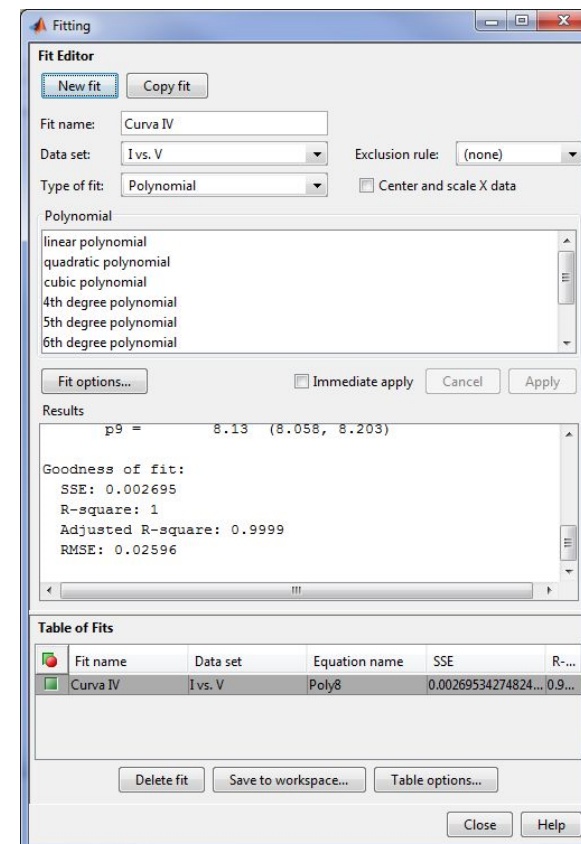
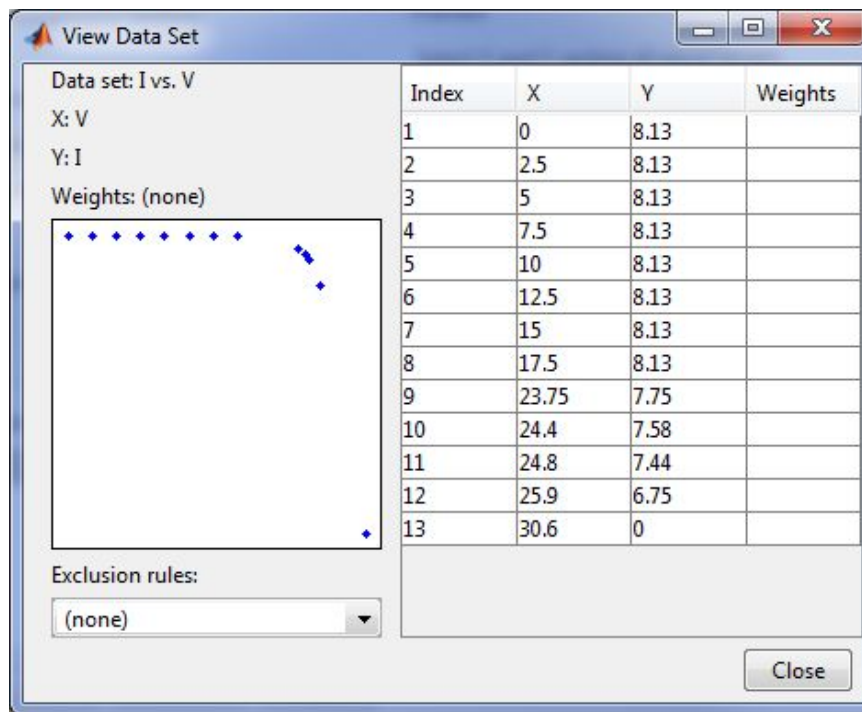




Matlab Tools: Curve Fitting Toolbox

Fitting Tab

Data Tab

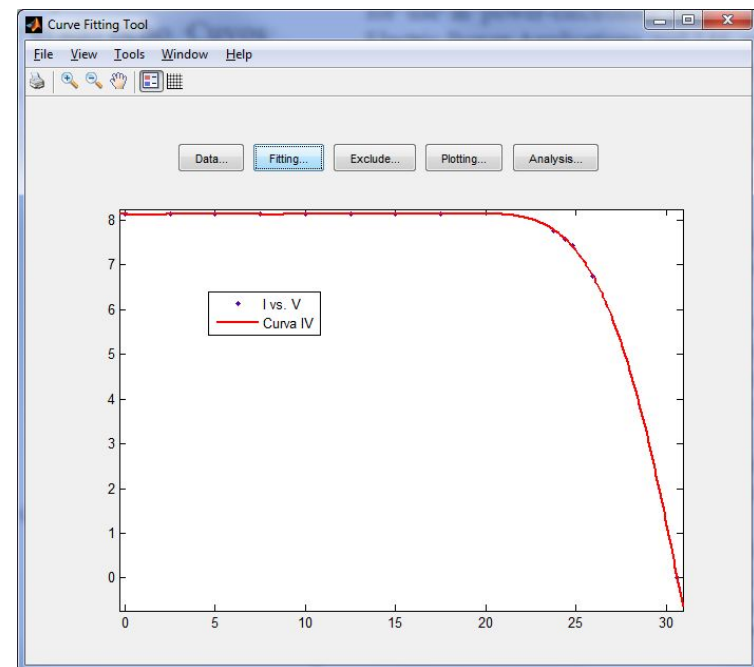




Matlab Tools: Curve Fitting Toolbox

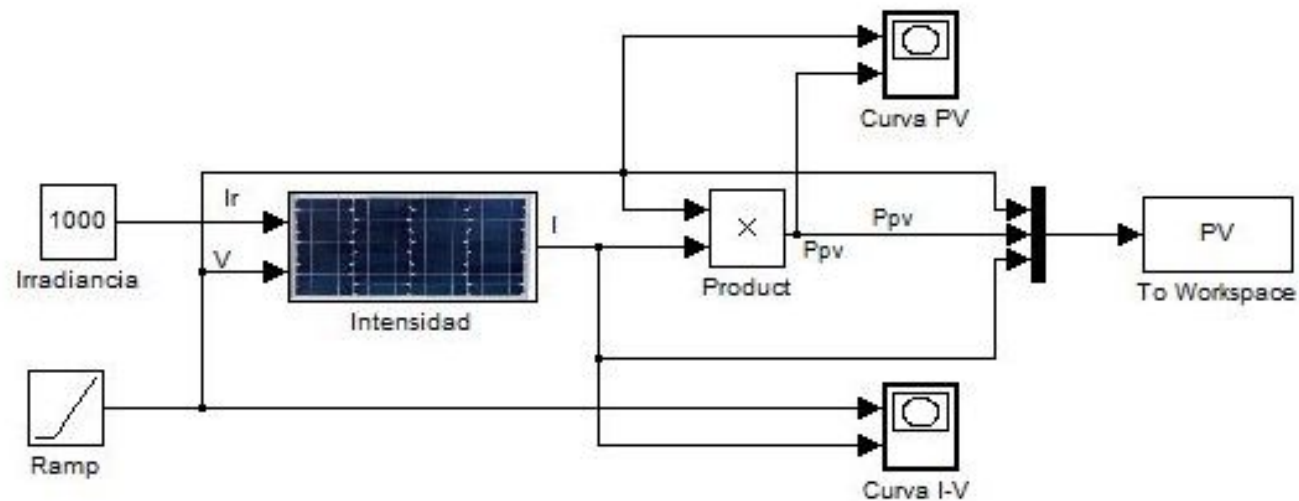
$$f(x) = p_1 * x^8 + p_2 * x^7 + p_3 * x^6 + p_4 * x^5 + p_5 * x^4 + p_6 * x^3 + p_7 * x^2 + p_8 * x + p_9$$

- $p_1 = 1.536e-009$
- $p_2 = -1.699e-007$
- $p_3 = 7.451e-006$
- $p_4 = -0.0001679$
- $p_5 = 0.002085$
- $p_6 = -0.01407$
- $p_7 = 0.04677$
- $p_8 = -0.05706$
- $p_9 = 8.13$





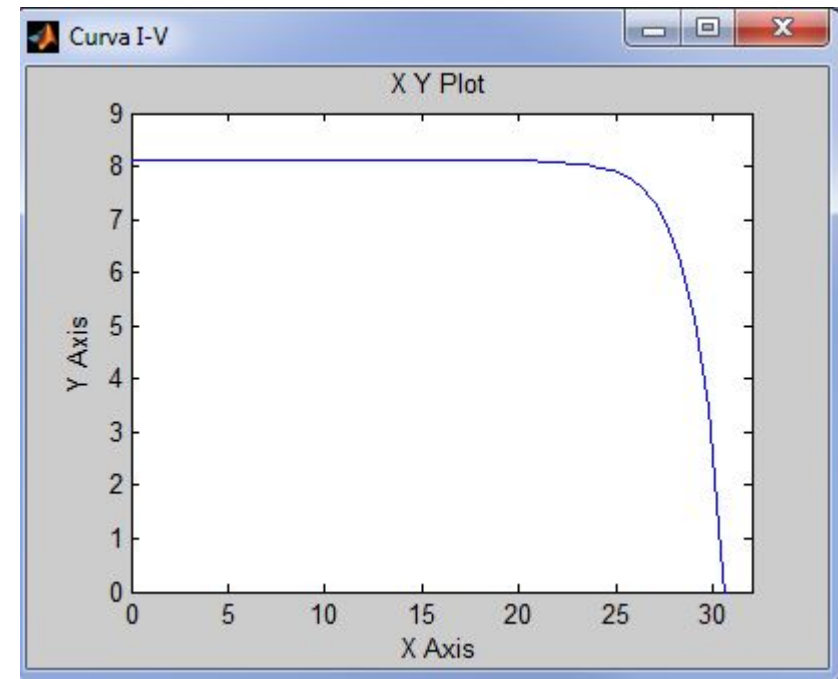
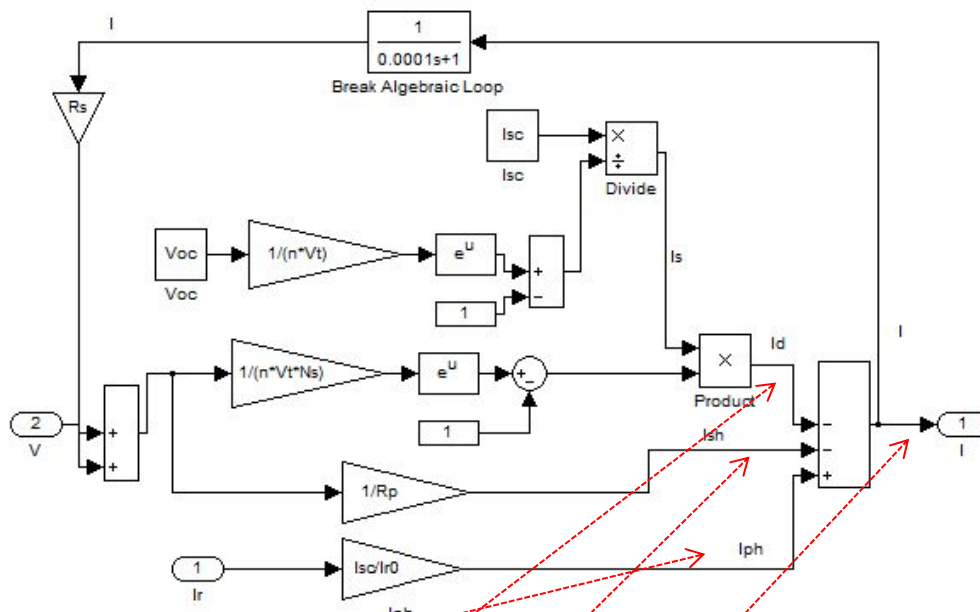
Basic Model in Simulink





Basic Model in Simulink

$$I = I_{ph} - I_s * (e^{(V+I*R_s)/N*V_t} - 1) - (V+I*R_s)/R_p$$



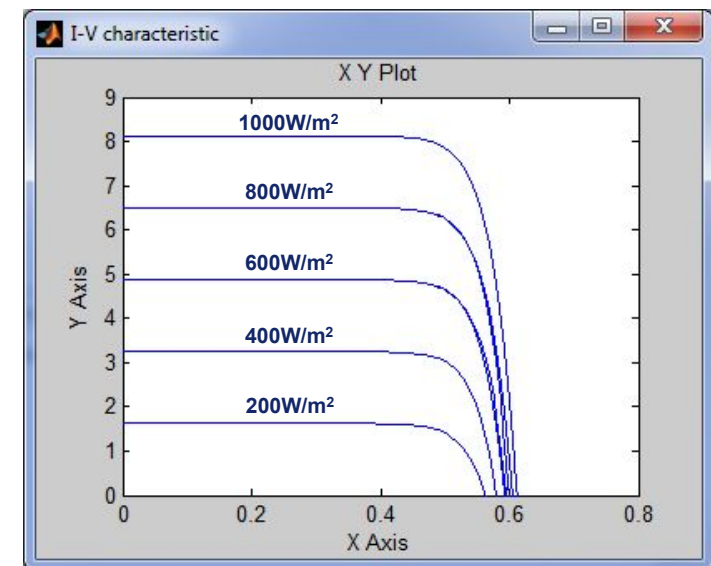
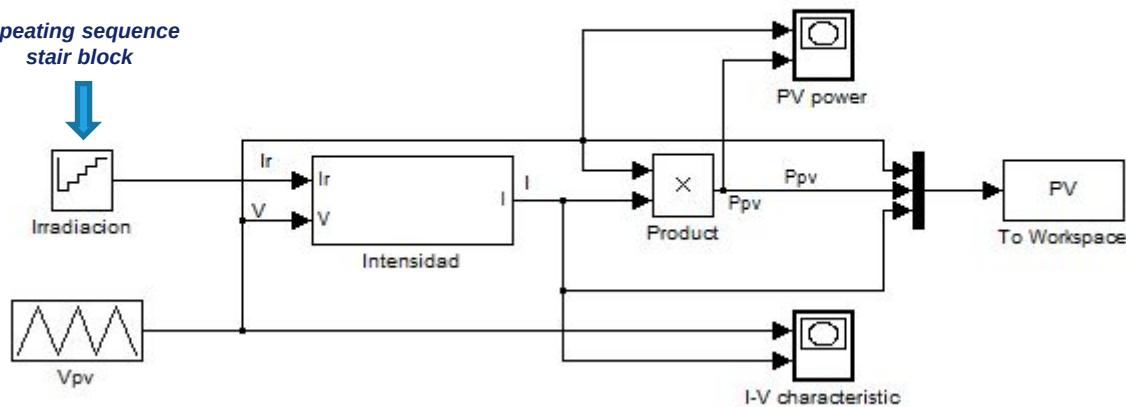
$$I_{ph} - I_d - I_{sh} = I$$



Basic Model in Simulink

$$I = I_{ph} - I_s * (e^{(V + I * R_s) / N * V_t} - 1) - (V + I * R_s) / R_p$$

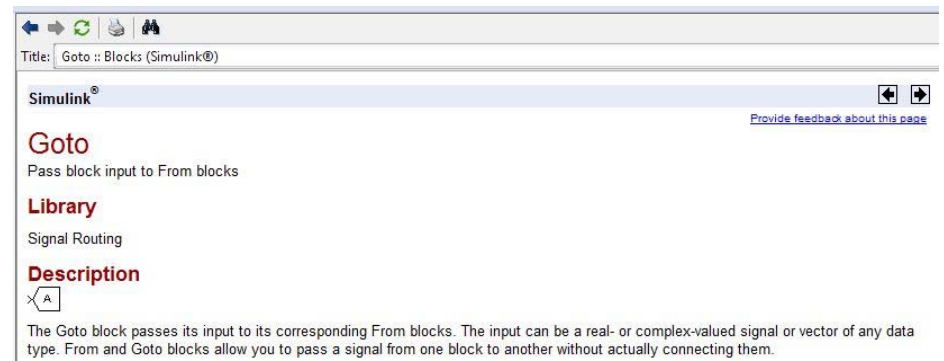
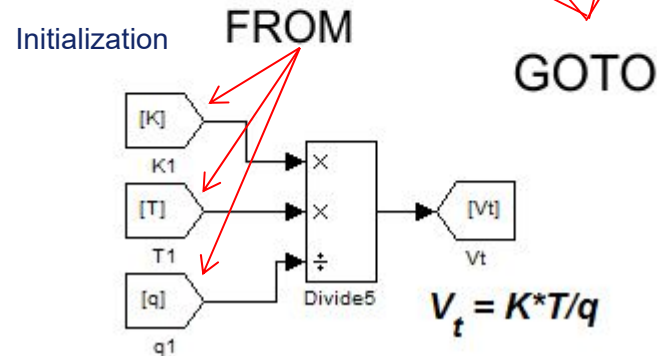
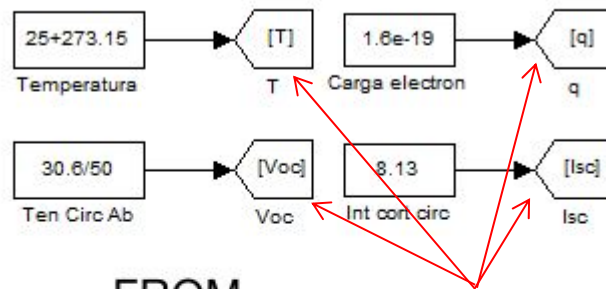
Repeating sequence stair block



Photovoltaic Panel

Simulink model with Tags

Initial conditions

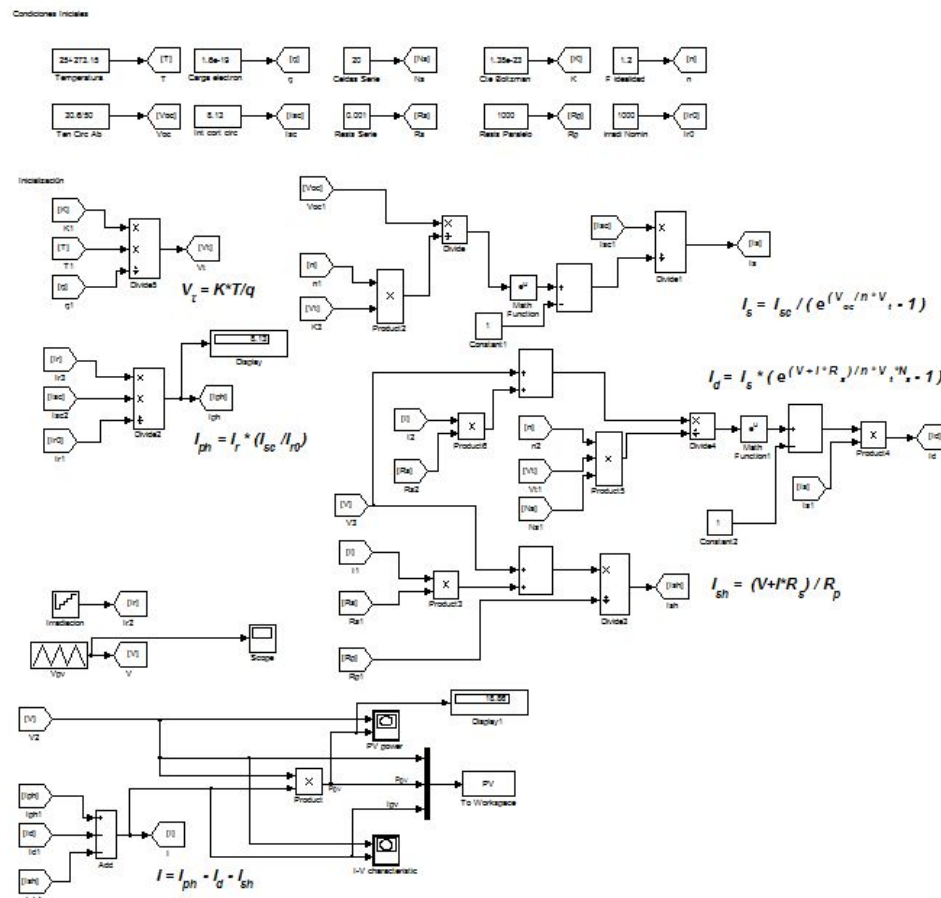


Photovoltaic Panel

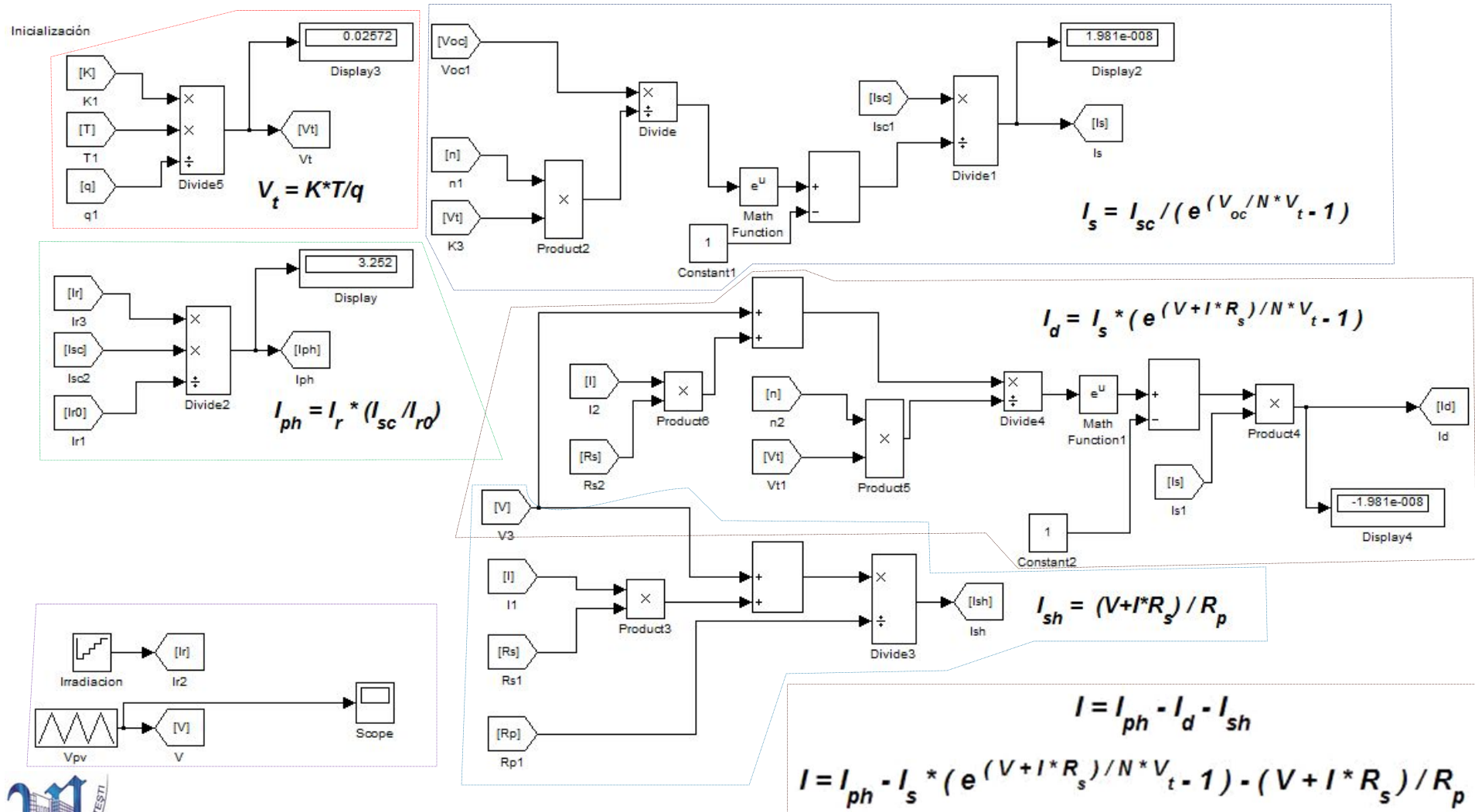
Simulink model with Tags

$$I = I_{ph} - I_d - I_{sh}$$

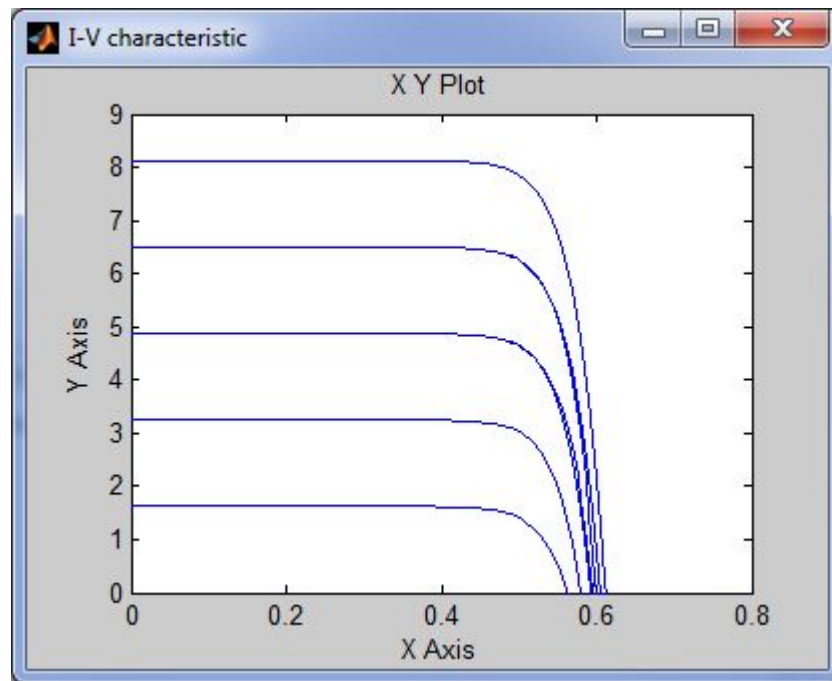
$$I = I_{ph} - I_s \cdot (e^{(V + I \cdot R_s) / (n \cdot V_t \cdot N_s)} - 1) \cdot (V + I \cdot R_s) / R_p$$



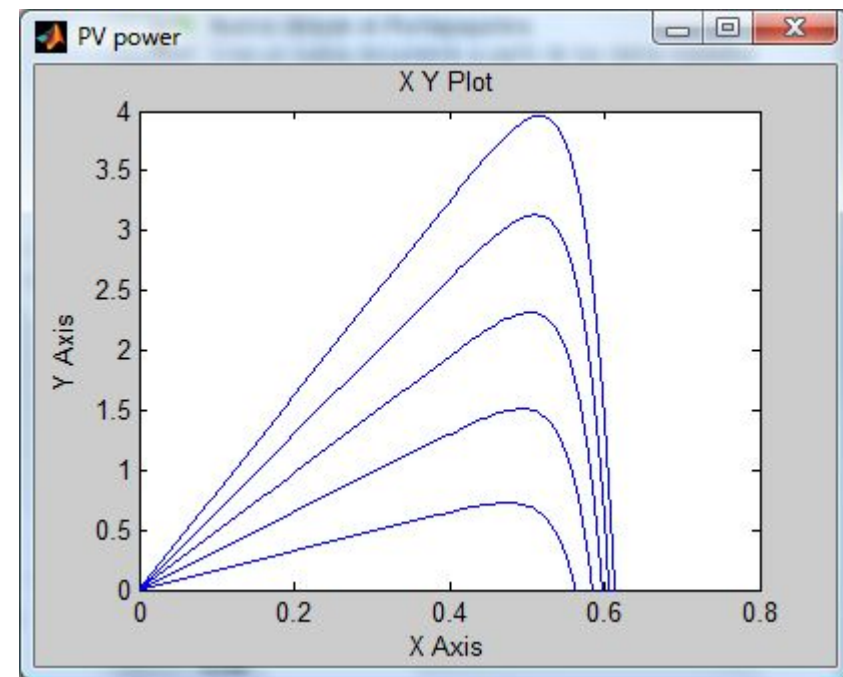
Photovoltaic Panel



Photovoltaic Panel



IV characteristic curves

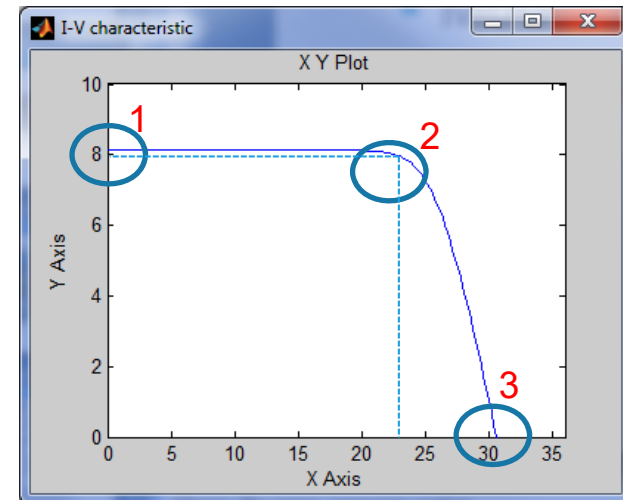


PV characteristic curves

Photovoltaic Panel

General equation :

$$I = I_L - I_0 \cdot \left(e^{\frac{(V + IR_s)q}{akT_cK}} - 1 \right) - \frac{(V + IR_s)}{R_{sh}}$$



We always know three points :

1. Short Circuit Current: $I = I_{sc}$, $V = 0$
2. Open Circuit Voltage: $I = 0$, $V = V_{oc}$
3. The maximum power point: $I = I_{mp}$, $V = V_{mp}$



Complex Mathematical Models

- ☀️ I_L : Photocurrent,.
- ☀️ I_0 : Reverse saturation current of the diode,
- ☀️ R_s : Series resistance,
- ☀️ R_p : Shunt resistance,
- ☀️ a : A parameter curve fit

🌍 Model of three parameters

🔋 $R_s, (R_{sh} \rightarrow \infty), a, (I_L = I_{sc})$ e I_0 .

🌍 Model of four parameters

🔋 $R_s, (R_{sh} \rightarrow \infty), a, I_L$ e I_0 .

🌍 Model of three parameters

🔋 R_s, R_{sh}, a, I_L e I_0 .



❖ Model of three parameters

Unknowns : R_s , ($R_{sh} \rightarrow \infty$), a , ($I_L = I_{sc}$) e I_0 .

General Equation:

$$I = I_L - I_0 \left(e^{\frac{V + IR_s}{a}} - 1 \right)$$

$$I_0 = \frac{I_{sc}}{\frac{V_{oc}}{V_T} e^{\frac{V_{oc}}{V_T}}}$$

$$R_s = \frac{V_{oc} - V_{mp} + V_T \cdot \ln\left(\frac{I_{sc} - I_{mp}}{I_{sc}}\right)}{I_{mp}}$$

$$a = \frac{\beta_{V_{oc}} T_c - V_{oc} + \varepsilon N_s}{\frac{\alpha_{I_{sc}} T_c}{I_L} - 3}$$



❖ Model of four parameters

Incógnitas: R_s , ($R_{sh} \rightarrow \infty$), a , I_L e I_0

General Equation:

$$I = I_L - I_0 \left(e^{\frac{V + IR_s}{a}} - 1 \right)$$

$$I = I_{sc} \text{ y } V = 0$$

$$I_{sc} = I_L - I_0 \left(e^{\frac{I_{sc} R_s}{a}} - 1 \right)$$

$$I = 0 \text{ y } V = V_{oc}$$

$$0 = I_L - I_0 \left(e^{\frac{V_{oc}}{a}} - 1 \right)$$

$$I = I_{mpp} \text{ y } V = V_{mpp}$$

$$I_{mpp} = I_L - I_0 \left(e^{\frac{V_{mpp} + I_{mpp} R_s}{a}} - 1 \right)$$

According to Townsend

$$a = \frac{\beta V_{oc} T_c - V_{oc} + \varepsilon N_s}{\frac{\alpha I_{sc} T_c}{I_L} - 3}$$



❖ Model of five parameters

$$I = I_L - I_0 \cdot \left(e^{\frac{(V + IR_s)q}{akT_{cK}}} - 1 \right) - \frac{(V + IR_s)}{R_{sh}}$$

- Short circuit current : $I = I_{sc}$, $V = 0$
- Open circuit voltage : $I = 0$, $V = V_{oc}$
- At the point of maximum power : $I = I_{mp}$, $V = V_{mp}$
- At the point of maximum power : $dP/dV_{mp} = 0$
- In short circuit: $dI_{sc} / dV = -1/R_{sh}$



❖ Model of five parameters

$$I = I_L - I_0 \cdot \left(e^{\frac{(V + I R_s)q}{akT_{cK}}} - 1 \right) - \frac{(V + I R_s)}{R_{sh}}$$

$$(1) \quad I_{sc} = I_{ph} - I_0 e^{\frac{I_{sc} R_s}{n_s V_T}} - \frac{I_{sc} R_s}{R_{sh}} \quad P(0, I_{sc})$$

$$(2) \quad I_{oc} = 0 = I_{ph} - I_0 e^{\frac{V_{oc}}{n_s V_T}} - \frac{V_{oc}}{R_{sh}} \quad P(V_{oc}, 0)$$

$$(3) \quad I_{mpp} = I_{ph} - I_0 e^{\frac{V_{mpp} + I_{mpp} R_s}{n_s V_T}} - \frac{V_{mpp} + I_{mpp} R_s}{R_{sh}} \quad P(V_{mp}, I_{mp})$$



❖ Model of five parameters

$$(4) \quad \left. \frac{dP}{dV} \right|_{I=I_{mpp}} = I_{mpp} + V_{mpp} \frac{\frac{-(I_{sc}R_{sh} - V_{oc} + I_{sc}R_s)e^{\frac{V_{mpp}+I_{mpp}R_s-V_{oc}}{n_sV_T}}}{n_sV_T R_{sh}} - \frac{1}{R_{sh}}}{1 + \frac{(I_{sc}R_{sh} - V_{oc} + I_{sc}R_s)e^{\frac{V_{mpp}+I_{mpp}R_s-V_{oc}}{n_sV_T}}}{n_sV_T R_{sh}} + \frac{R_s}{R_{sh}}} \quad \frac{dP}{dV}_{mp} = 0$$

$dI_{sc} / dV = -1/R_{sh}$

$$(5) \quad \left. -\frac{1}{R_{sh}} \right|_{I=I_{sc}} = \frac{\frac{-(I_{sc}R_{sh} - V_{oc} + I_{sc}R_s)e^{\frac{I_{sc}R_s-V_{oc}}{n_sV_T}}}{n_sV_T R_{sh}} - \frac{1}{R_{sh}}}{1 + \frac{(I_{sc}R_{sh} - V_{oc} + I_{sc}R_s)e^{\frac{I_{sc}R_s-V_{oc}}{n_sV_T}}}{n_sV_T R_{sh}} + \frac{R_s}{R_{sh}}} \quad dI_{sc} / dV = -1/R_{sh}$$

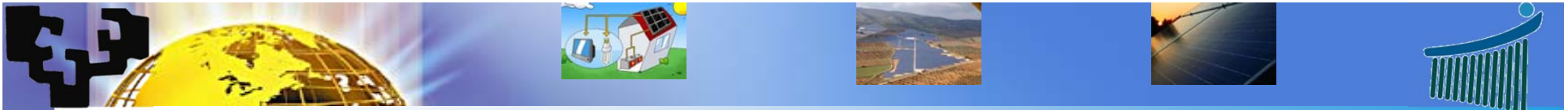


❖ Full model

$$I = I_L - I_0 \cdot \left(e^{\frac{(V + I R_s)q}{akT_{cK}}} - 1 \right) - \frac{(V + I R_s)}{R_{sh}}$$

$$I_{sc,ref} = I_{L,ref} - I_{0,ref} \left(e^{\frac{I_{sc,ref} R_{s,ref}}{a_{ref}}} - 1 \right) - \frac{I_{sc,ref} R_{s,ref}}{R_{sh,ref}}$$

$$0 = I_{L,ref} - I_{0,ref} \left(e^{\frac{V_{oc,ref}}{a_{ref}}} - 1 \right) - \frac{V_{oc,ref}}{R_{sh,ref}}$$



Panel Fotovoltaico

❖ Full model

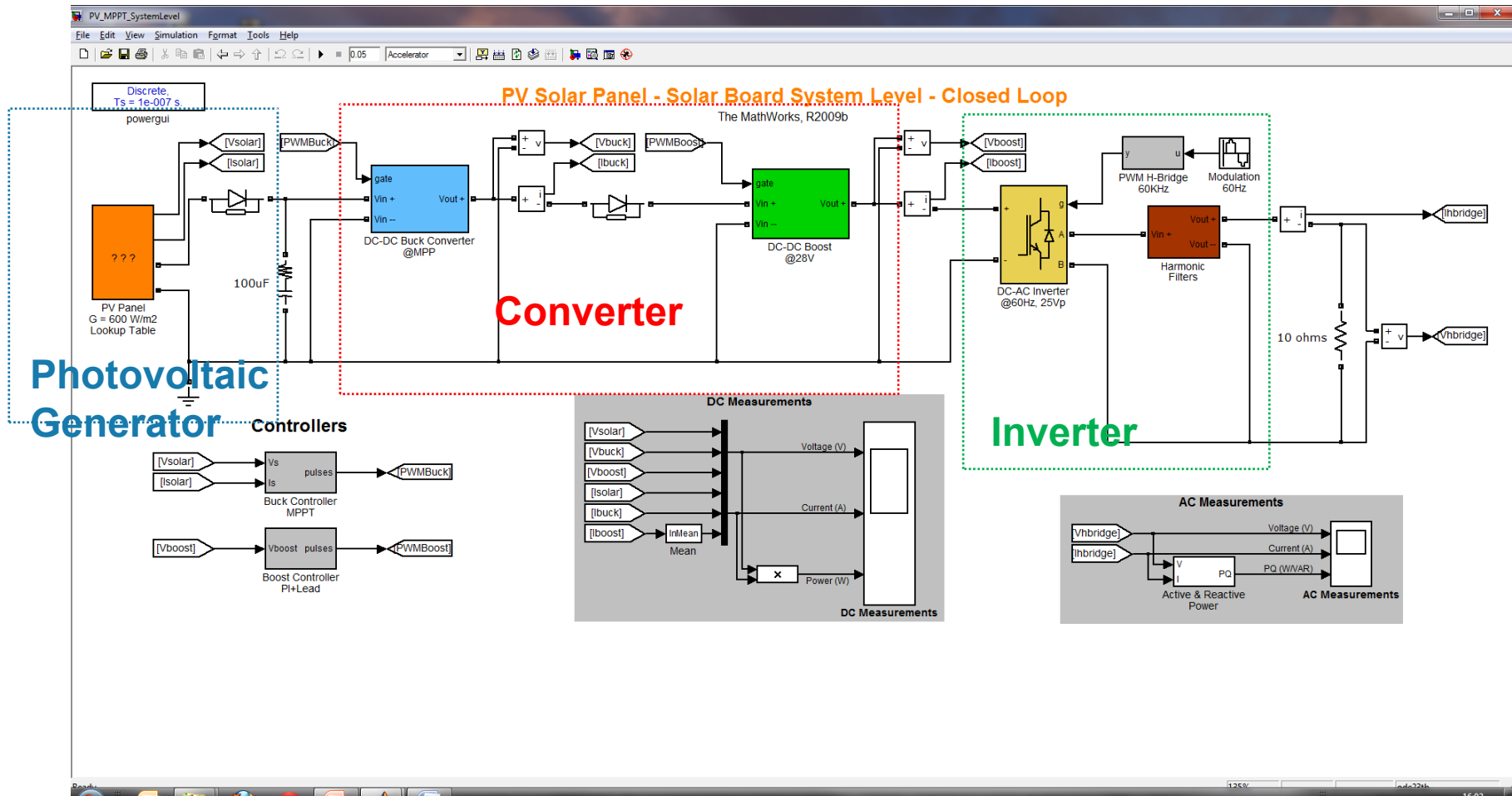
$$I_{mp,ref} = I_{L,ref} - I_{0,ref} \left(e^{\frac{V_{mp,ref} + I_{mp,ref} R_{s,ref}}{a_{ref}}} - 1 \right) - \frac{V_{mp,ref} + I_{mp,ref} R_{s,ref}}{R_{sh,ref}}$$

$$\left. \frac{d(IV)}{dV} \right|_{mp} = I_{mp} - V_{mp} \left. \frac{dI}{dV} \right|_{mp} = 0$$

$$\left. \frac{dI}{dV} \right|_{mp} = \frac{\frac{-I_0}{a} e^{\frac{V_{mp} + I_{mp} R_s}{a}} - \frac{1}{R_{sh}}}{1 + \frac{I_0 R_s}{a} e^{\frac{V_{mp} + I_{mp} R_s}{a}} + \frac{R_s}{R_{sh}}}$$

$$\beta_{V_{oc}} = \left. \frac{\partial V}{\partial T} \right|_{I=0} \approx \frac{V_{oc,ref} - V_{oc,T_c}}{T_{ref} - T_c}$$

Photovoltaic System





Photovoltaic System

❖ Types of DC / DC converters basic

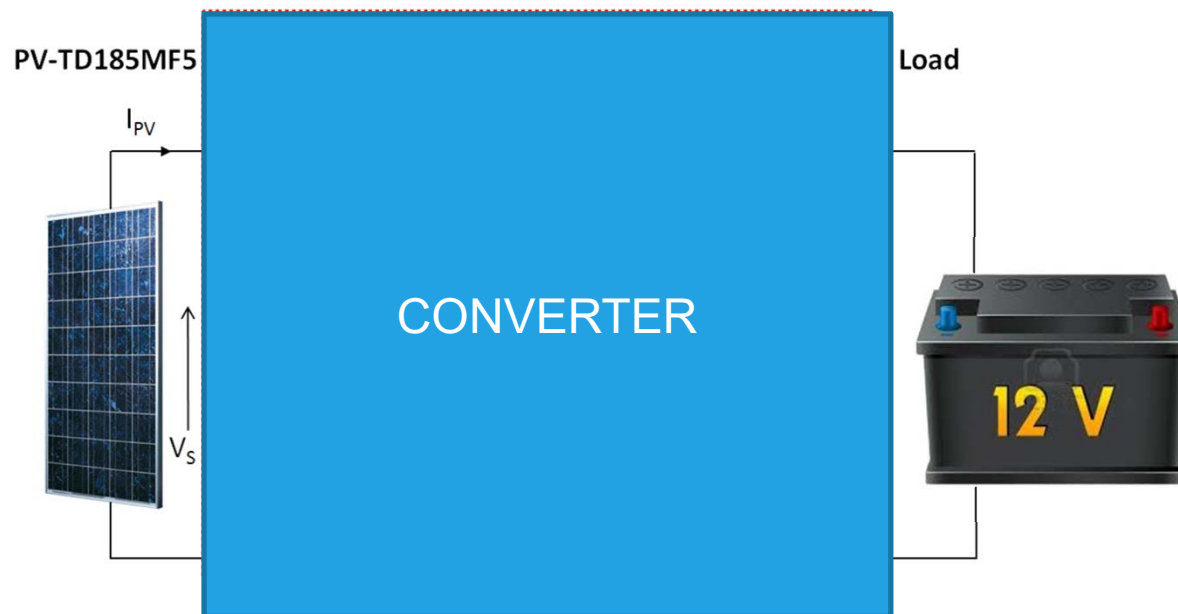
➤ **Boost converter:** $V_{\text{Load}} > V_{\text{Module}}$

➤ **Buck converter:** $V_{\text{Load}} < V_{\text{Module}}$

➤ **Buck-Boost converter:** $V_{\text{Load}} > V_{\text{Module}}$ or $V_{\text{Load}} < V_{\text{Module}}$

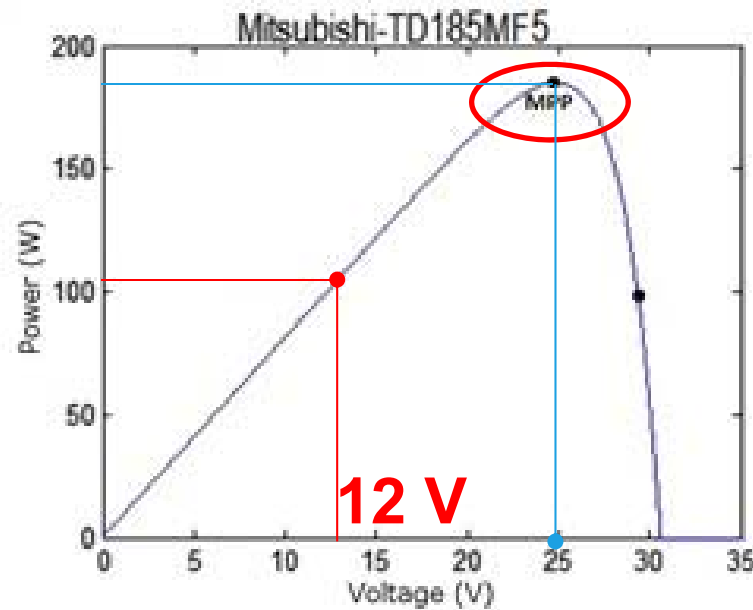
Photovoltaic System

❖ Why do we need to put a converter, for example between the photovoltaic module and battery? For example.



Photovoltaic System

- ❖ Why do we need to put a converter, for example between the photovoltaic module and battery?

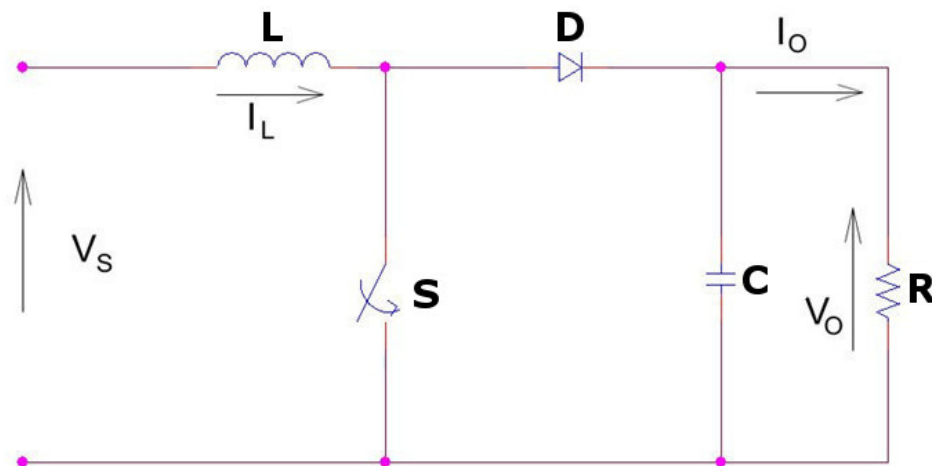


For $G = 1000 \text{ W/m}^2$
 $T = 25 \text{ }^{\circ}\text{C}$

Our efficiency is poor. We need to work near the MPP

Photovoltaic System

❖ Types of DC / DC converters basic



Boost converter

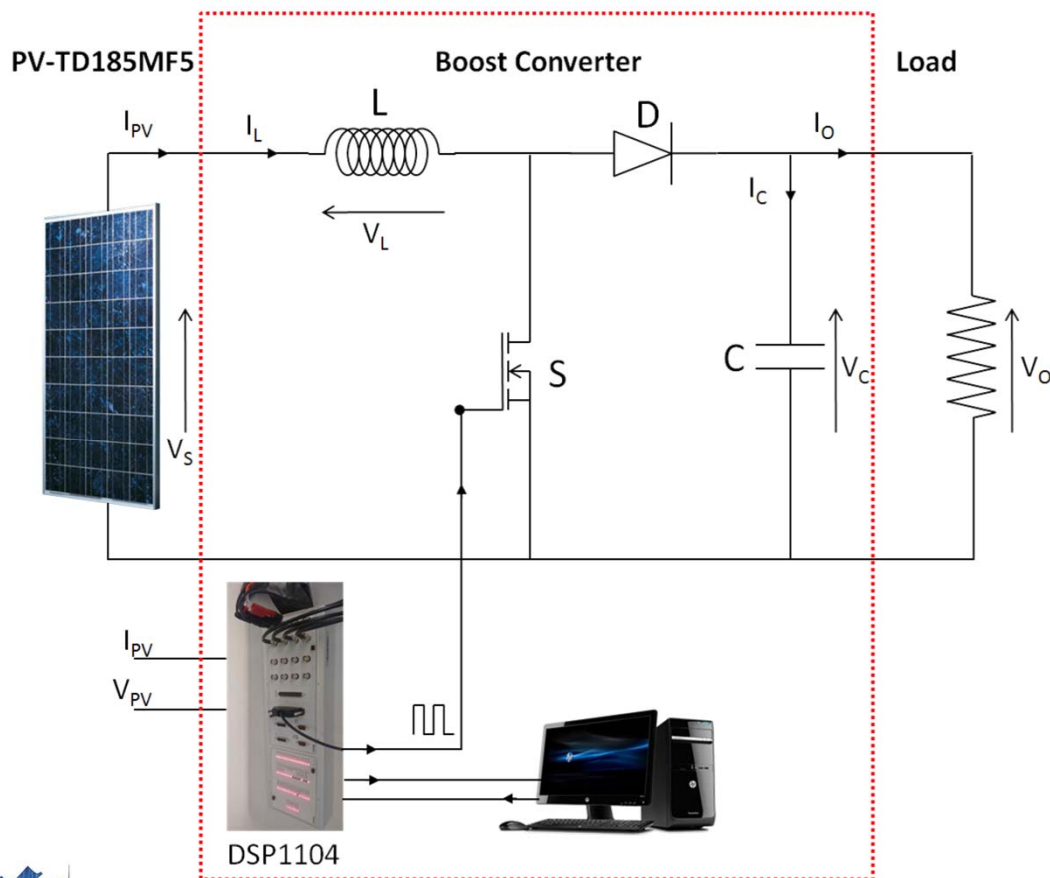
In the case of a Boost converter in continuous driving operating mode, the relations between the input (V_s) and output (V_o) variables are given by Equations

$$V_{\text{Load}} > V_{\text{Module}}$$

$$V_o = \frac{V_s}{(1 - \delta)}$$

Photovoltaic System

❖ How do we do it:



$$V_o = \frac{V_s}{(1 - \delta)}$$

With de MPPT look for δ

$$\delta = 1 - \frac{V_s}{V_o}$$